Let V be a finite dimensional inner product space with inner product $\langle \cdot, \cdot \rangle$. Suppose that $P : V \to V$ is linear, satisfies $P^2 = P$ and also satisfies $\langle Pv, w \rangle = \langle v, Pw \rangle$ for all $v, w \in V$. Prove that there exists a subspace $W \subset V$ such that $P = P_W$, i.e. P is an orthogonal projection onto subspace W.