We've seen the formula that for W_1, W_2 finite-dimensional subspaces of a vector space V, we have that

$$\dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$$

Recall the famed **principle of inclusion-exclusion** for finite sets, A, B. It states that $\#(A \cup B) = \#A + \#B - \#(A \cap B)$. Of course, the principle of inclusion-exclusion generalizes to account for multifold intersections. It's general form takes the *very scary* looking form

$$\#(\cap_{i=1}^{n} A_i) = \sum_{k=1}^{n} (-1)^{k+1} \{ \sum_{1 \leq i_1 < i_2 < \cdots i_k \leq n} \#(\cap_{j=1}^{k} A_{i_j}) \}$$

If we specify to $n = 3$ we get the principle of inclusion-exclusion for three sets which takes the form

$$\#(A \cup B \cup C) = \#A + \#B + \#C - \#(A \cap B) - \#(A \cap C) - \#(B \cap C) + \#(A \cap B \cap C)$$

So, one might then ask whether it is true or not that the following

$$\begin{aligned} \dim(W_1 + W_2 + W_3) &= \dim(W_1) + \dim(W_2) + \dim(W_3) \\ &\quad - \dim(W_1 \cap W_2) - \dim(W_1 \cap W_3) - \dim(W_2 \cap W_3) + \dim(W_1 \cap W_2 \cap W_3) \end{aligned}$$

holds for all finite-dimensional subspaces W_1, W_2, W_3 of a real vector space V? Your job is to prove the boxed formula or find a counterexample.