MAT224 Final Exam Proof Clinics

In preparation for the upcoming final exam we are once again holding proof clinics. These sessions will occur at the following times and locations:

Date	Time	Location
Friday, Mar. 29	12:00 pm - 2:00 pm	RW 143
Saturday, Mar. 30	11:00 am - 1:00 pm	SS 1088
Sunday, Mar. 31	11:00 am - 1:00 pm	BA 2135
Monday, Apr. 1	11:00 am - 1:00 pm	UC 261
Tuesday, Apr. 2	11:00 am - 1:00 pm	FE 135
Wednesday, Apr. 3	11:00 am - 1:00 pm	FE 24
Thursday, Apr. 4	11:00 am - 1:00 pm	GB 220
Friday, Apr. 5	12:00 pm - 2:00 pm	RW 143
Saturday, Apr. 6	11:00 am - 1:00 pm	SS 1074
Sunday, Apr. 7	11:00 am - 1:00 pm	BA 2135

These sessions will be the last ones before the final exam so be sure to attend if you want personal feedback from a TA regarding your proofs. Five exercises are given below and additional exercises will be provided during the sessions to simulate an exam environment. As was the case for past sessions, we suggest having complete proofs to the exercises below before attending.

In the problems below V is a finite-dimensional vector space.

1. If $A \in M_{n\times n}(\mathbb{C})$ satisfies $A^3 = 2I$, show that $B = A^2 - 2A + 2I$ is invertible.
2. Let $A, B \in M_{n\times n}(\mathbb{C})$ and let $p_B(x)$ be the characteristic polynomial of B. Show that the matrix $p_B(A)$ is invertible if and only if A and B have no common eigenvalues.
3. Let $A, B \in M_{n\times n}(\mathbb{C})$. Determine whether the following statements are true or false. If true, prove the statement. If false, provide a counterexample.
   (a) If $A^k = 0$ for all integers $k \geq 2$ then $A = 0$.
   (b) If $A^k = 0$ for some integer $k \geq 2$ then $\operatorname{tr}(A) = 0$.
   (c) If $A^k = 0$ for some integer $k \geq 2$ then $\det(A) = 0$.
   (d) If $\operatorname{rank} A = r$ then A has r nonzero eigenvalues, counting multiplicity.
   (e) If A has r nonzero eigenvalues counting multiplicity then $\operatorname{rank} A \geq r$.
   (f) If A and B are similar then A and B have the same eigenvalues.
   (g) If A and B have the same eigenvalues then A and B are similar.
4. Let $T\colon V \to V$ be a linear transformation and let $V_1, V_2 \subseteq V$ be subspaces. Prove or disprove the following:

(a) $T(V_1 \cap V_2) = T(V_1) \cap T(V_2)$

(b) $T(V_1 \cup V_2) = T(V_1) \cup T(V_2)$

(c) $T(V_1 + V_2) = T(V_1) + T(V_2)$

(d) If $V_1 \cap V_2 = \{0\}$ then $T(V_1 \oplus V_2) = T(V_1) \oplus T(V_2)$

5. Suppose V is an inner product space and let $y, z \in V$ be such that for all $x \in V$, $\langle x, y \rangle = \langle x, z \rangle$. Show that $y = z$.