MAT224 Proof Clinics

To help prepare you for the upcoming midterm we will be holding proof clinics. There will be sessions at the following times:

Date	Time	Location
Saturday, Feb. 2	1:00 pm - 3:00 pm	BA 1220
Sunday, Feb. 3	1:00 pm - 3:00 pm	BA 1220
Monday, Feb. 4	11:00 am - 1:00 pm	UC 140
Tuesday, Feb. 5	10:00 am - 12:00 pm	MP 137

These sessions are your chances to get personal feedback from a TA on writing proofs so we strongly recommend you attend! We have prepared a list of exercises, five of which are given below. The rest of the exercises will be given during these sessions to simulate a test environment and gauge your ability to construct a proof on the spot. To make the most of these sessions we highly recommend showing up with written proofs for the exercises below.

1. Let V be a real vector space with zero vector $\mathbf{0} \in V$.
   (a) Show that for all $c \in \mathbb{R}$ we have $c \cdot \mathbf{0} = \mathbf{0}$.
   (b) Show that if $c \in \mathbb{R}$ and $v \in V$ are such that $cv = \mathbf{0}$ then $c = 0$ or $v = \mathbf{0}$.
2. Suppose V is a vector space with subspaces $W_1, W_2 \subseteq V$. Show that $W_1 \cup W_2$ is a subspace of V if and only if $W_1 \subseteq W_2$ or $W_2 \subseteq W_1$.
3. Suppose V is a vector space and $v_1, v_2, v_3 \in V$ are such that $v_1 + v_2 + v_3 = 0$. Show that $\operatorname{span}\{v_1, v_2\} = \operatorname{span}\{v_2, v_3\}$.
4. Let V be a vector space with subspaces $W_1, W_2, W_3 \subseteq V$.
   (a) Prove that $(W_1 \cap W_2) + W_3 \subseteq (W_1 + W_3) \cap (W_2 + W_3)$.
   (b) Provide an example illustrating that this is not always an equality.
5. Suppose V and W are finite-dimensional vector spaces. We can define a new vector space from V and W, called *the direct product of V and W*, as

   $$V \times W = \{(v, w) \colon v \in V, w \in W\}$$

   Given $(v_1, w_1), (v_2, w_2) \in V \times W$ and $c \in \mathbb{R}$ addition is defined as $(v_1, w_1) + (v_2, w_2) = (v_1 + v_2, w_1 + w_2)$ and scalar multiplication as $c(v_1, w_1) = (cv_1, cw_1)$.
   (a) Verify that $V \times W$ is indeed a vector space.
   (b) If $\mathbf{0}_V \in V$ and $\mathbf{0}_W \in W$ are the zero vectors of V and W respectively then what is the zero vector of $V \times W$? Prove your claim.
   (c) What is $\dim V \times W$ in terms of $\dim V$ and $\dim W$? Prove your claim.