MAT224 Warm-Up

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1 Instructions

What follows are some main results you should know from a course like MAT223. We expect you to be familiar with the results *and to be able to provide proofs*. Go through these problems before the first lecture, and use it as a check on material you may wish to review. It will be way better for you to master this material at the beginning of term rather than once the MAT224 material becomes more advanced. If you're stuck on these, please make use of the course Piazza forum. If you need a refresher on terms/ideas in linear algebra, I recommend "Linear Algebra and its Applications" by Lay, Lay, and McDonald, 5th edition.

2 Problems

1. Prove that, surprisingly, 2 planes in $\mathbb{R}^4$ cannot meet in a line.

2. Let A, B be compatible matrices. Prove that $rank(AB) \leq \min(rank(A), rank(B))$.

3. Let $\mathbf{v}_1, ..., \mathbf{v}_n$ be eigenvectors of a matrix A corresponding to *distinct* eigenvalues $\lambda_1, ..., \lambda_n$. Prove that $\mathbf{v}_1, ..., \mathbf{v}_n$ are linearly independent.

4. Let $A \in M_{m\times n}$ be an $m\times n$ matrix with rank n. Prove that there exists an $n\times m$ matrix C satisfying $CA = I$, where I is the $n \times n$ identity matrix.

5. Diagonalize $\begin{bmatrix} 2 & 2 & 4 \\ 2 & 2 & 4 \\ 2 & 2 & 4 \end{bmatrix}$.

6. **Instructions:** *Below, I attempt to prove a "theorem" which is actually not true. Find a mistake in the argument and explain why it is mistaken.*
   "Theorem". *If A, B are compatible matrices and AB is invertible, then so must be A and B*
   "Proof". If AB is invertible then $\det(AB) \neq 0$. But $\det(AB) = \det(A)\det(B)$ so therefore $\det(A)\det(B) \neq 0$. But the product of two numbers is nonzero if and only if each of the numbers is non-zero, i.e. $\det(A) \neq 0$ and $\det(B) \neq 0$. Thus, A and B are invertible. QED.

7. Give a counterexample to the false "Theorem" given in the preceding question.