Instructions: READ THIS

Here's how I recommend using this mock midterm.

(i) Find a day where you can set aside 50 minutes at the scheduled exam time (i.e. 3:10-4:00 PM).

(ii) Set a timer for 50 minutes.

(iii) Go through the exam as you would on the test date. Simulate test conditions as accurately as possible.

(iv) Afterwards, grade yourself (you'll have to estimate reasonable point deductions - **don't grade yourself gently**).

(v) On places you missed points, use this as an opportunity to review selected material.

(vi) If you found yourself struggling to finish in the allowed time this indicates you're not as solid on the material as you might have thought.

(vii) It might be nice to take it with a friend or a study group and discuss your results with each other at the end.

Now, here's how **NOT** to use this mock midterm.

(i) **Don't** flip through it before sitting and taking it (i.e. no peeking!) You're really only hurting yourself by doing this.

(ii) **Don't** take it and go easy on yourself either with time or with grading.

(iii) **Don't** view this as any assurance of the difficulty of the actual test. I am aiming for similarity in style and a rough approximation of difficulty but, for what I hope are obvious reasons, this isn't ever actually possible.

(iv) **Don't** use your (presumed excellent) performance on this as a reason to become complacent. There's a lot of material from which you can be tested so there's plenty of further opportunity to be challenged on the test day.

FACULTY OF ARTS & SCIENCES
University of Toronto

MAT224: Linear Algebra II

PRACTICE Midterm Exam #2

Duration: 50 minutes

Total: 86 marks

Family Name: _____

(Please Print- You will lose 3 points for getting this wrong)

Given Name(s): _____

(Please Print)

Student Number: _____

Toronto Email: _____

Signature: _____

You may NOT use calculators, or any electronic devices during the test. You must completely justify your answers. Do NOT remove any pages from the test booklet.

FOR MARKER'S USE ONLY			
Problem 1:	/20	Problem 2:	/10
Problem 3:	/15	Problem 4:	/20
Problem 5:	/21		
		TOTAL:	/86

Problem 1. (20 points, 10 points each).

   (i) Given an $n \times n$ matrix A, define the **characteristic polynomial** of A.

   (ii) State the dimension theorem.

Problem 2. (10 points). Let $T : P_3(\mathbb{R}) \to P_3(\mathbb{R})$ be the linear map defined by $T(p)(x) = x^2 p''(x)$. Find all eigenvalues and eigenvectors of T. Is T diagonalizable? Why or why not?

Problem 3. (15 points). Let $T : V \to V$ be a linear map on real vector space V. Suppose that T^2 is injective. Prove that T must be injective.

Problem 4. (20 points). Let V be a finite dimensional real vector space. If $S, T \in \mathcal{L}(V, V)$ prove that ST and TS have the same eigenvalues whenever T is an isomorphism. (*Actually, the result is true even when T is not an isomorphism, so your proof need not depend on this fact, it's just one way to prove this is easier if you make this assumption.*)

Problem 5. (3 points for each correct answer, minus 2 points for each incorrect answer for a minimum of 0 points or a maximum of 21 points). For the following questions, answer using the word "True" or the word "False". You **don't need to justify your answer** to receive full credit. There's no partial credit.

(i) True/False: Invertible linear transformations are diagonalizable.

(ii) True/False: Isomorphisms are both injective and surjective.

(iii) True/False: If A is $n \times n$ and $\det(A) = 0$ then the characteristic polynomial $p_A(t) = t^k q(t)$ for some $k \in \mathbb{N}$ and $q(0) \neq 0$.

(iv) True/False: Let V and W be finite dimensional real vector spaces. If $T : V \to W$ is linear and invertible, then $\dim(V) = \dim(W)$.

(v) True/False: Suppose $T : V \to W$ is linear, for real vector spaces, and $\dim(V) = 3$, $\dim(W) = 2$. Then $\dim(\ker(T))$ can equal 0.

(vi) True/False: Similar matrices are isomorphic.

(vii) True/False: Let A, B be $n \times n$ similar matrices. Then $rank(A) = rank(B)$.

(*extra paper*)