Instructions: READ THIS

Here's how I recommend using this mock midterm.

(i) Find a day where you can set aside 50 minutes at the scheduled exam time (i.e. 3:10-4:00 PM).

(ii) Set a timer for 50 minutes.

(iii) Go through the exam as you would on the test date. Simulate test conditions as accurately as possible.

(iv) Afterwards, grade yourself (you'll have to estimate reasonable point deductions - **don't grade yourself gently**).

(v) On places you missed points, use this as an opportunity to review selected material.

(vi) If you found yourself struggling to finish in the allowed time this indicates you're not as solid on the material as you might have thought.

(vii) It might be nice to take it with a friend or a study group and discuss your results with each other at the end.

Now, here's how **NOT** to use this mock midterm.

(i) **Don't** flip through it before sitting and taking it (i.e. no peeking!) You're really only hurting yourself by doing this.

(ii) **Don't** take it and go easy on yourself either with time or with grading.

(iii) **Don't** view this as any assurance of the difficulty of the actual test. I am aiming for similarity in style and a rough approximation of difficulty but, for what I hope are obvious reasons, this isn't ever actually possible.

(iv) **Don't** use your (presumed excellent) performance on this as a reason to become complacent. There's a lot of material from which you can be tested so there's plenty of further opportunity to be challenged on the test day.

FACULTY OF ARTS & SCIENCES
University of Toronto

MAT224: Linear Algebra II

PRACTICE Midterm Exam #1

Duration: 50 minutes

Total: 65 marks

Family Name: ______________________________
(Please Print- You will lose 3 points for getting this wrong)

Given Name(s): ______________________________
(Please Print)

Student Number: ______________________________

Toronto Email: ______________________________

Signature: ______________________________

You may NOT use calculators, or any electronic devices during the test. You must completely justify your answers. Do NOT remove any pages from the test booklet.

FOR MARKER'S USE ONLY			
Problem 1:	/20	Problem 2:	/10
Problem 3:	/10	Problem 4:	/15
Problem 5:	/15	Problem 6:	/21
		TOTAL:	/91

Problem 1. (20 points). A matrix A is said to be *anti-symmetric* if $A^T = -A$. Consider the set $S = \{n \times n \text{ anti-symmetric matrices}\} \subset M_{n\times n}(\mathbb{R})$.

(i) (5 points). Prove that S is a subspace of $M_{n\times n}(\mathbb{R})$.

(ii) (15 points). Calculate $\dim(S)$ when $n = 3$ by explicitly providing a basis for S.

Problem 2. (10 points). Find all values k such that the set $S = \{x^3+x+1, x^3-x^2+1, x^3+kx^2+kx\}$ is linearly independent in $P_3(\mathbb{R})$.

Problem 3. (10 points). Let $T : P_2(\mathbb{R}) \to P_2(\mathbb{R})$ be a linear transformation satisfying $T(x+x^2) = 2x$, $T(1+x) = x^2$ and $T(1) = 1$. Find $T(2x^2 + x)$.

Problem 4. (15 points). Let U, V be subspaces of $P_2(\mathbb{R})$ defined by

$$U = \operatorname{span}\{1 + x + x^2, 2 + 4x + 6x^2\}, \qquad V = \operatorname{span}\{2 + 2x + 3x^2, 2 + 2x + 4x^2\}$$

Find $\dim(U \cap V)$.

Problem 5. (15 points) Let S_V be the set of all subspaces of a given vector space V. Define two binary operations $+, \cdot$ in S_V so that for $W_1, W_2 \in S_V$ then $W_1 + W_2$ has it's standard meaning and $cW_1 \doteq \{cw \mid w \in W_1\}$ when $c \in \mathbb{R}$. Is the set S_V, equipped with these operations, a vector space? If so, prove it, if not explain why not.

Problem 6. (3 points for each correct answer, minus 2 points for each incorrect answer for a minimum of 0 points or a maximum of 21 points). For the following questions, answer using the word "True" or the word "False". You **don't need to justify your answer** to receive full credit. There's no partial credit.

(i) True/False: Let V be a real vector space. Unions of subspaces of V are subspaces of V.

(ii) True/False: Let $S = \{x_1, ..., x_n\} \subset V$ where V is an n-dimensional real vector space. If $\text{span}(S) = V$ then S is linearly independent.

(iii) True/False: $\{f \mid \pi f'(x) + e^{\pi^e} f(x) = 0\}$ is a subspace of $C^1(\mathbb{R})$.

(iv) True/False: $C(\mathbb{R}) = U_e \oplus U_o$ for U_e, U_o odd and even continuous functions respectively.

(v) True/False: If $T : V \to W$ for real vector spaces V, W satisfies $T(0_V) = 0_W$ and $T(-v) = -T(v)$ for all $v \in V$ then T is a linear transformation.

(vi) True/False: if U is a subspace of real vector space V and $u \in U$ then $u + v \in U$ if and only if $v \in U$.

(vii) True/False: Every finite dimensional vector space has a basis.

(*extra paper*)