Instructions: READ THIS

Here's how I recommend using this mock midterm.

(i) Find a day where you can set aside 3 hours at the scheduled exam time (i.e. 7:00-10:00 PM).

(ii) Set a timer for 3 hours.

(iii) Go through the exam as you would on the test date. Simulate test conditions as accurately as possible.

(iv) Afterwards, grade yourself (you'll have to estimate reasonable point deductions - **don't grade yourself gently**).

(v) On places you missed points, use this as an opportunity to review selected material.

(vi) If you found yourself struggling to finish in the allowed time this indicates you're not as solid on the material as you might have thought.

(vii) It might be nice to take it with a friend or a study group and discuss your results with each other at the end.

Now, here's how **NOT** to use this mock midterm.

(i) **Don't** flip through it before sitting and taking it (i.e. no peeking!) You're really only hurting yourself by doing this.

(ii) **Don't** take it and go easy on yourself either with time or with grading.

(iii) **Don't** view this as any assurance of the difficulty of the actual test. I am aiming for similarity in style and a rough approximation of difficulty but, for what I hope are obvious reasons, this isn't ever actually possible.

(iv) **Don't** use your (presumed excellent) performance on this as a reason to become complacent. There's a lot of material from which you can be tested so there's plenty of further opportunity to be challenged on the test day.

FACULTY OF ARTS & SCIENCES
University of Toronto

MAT224: Linear Algebra II

PRACTICE Final Exam

Duration: 180 minutes

Total: 200 marks

Family Name: ________________________________
(Please Print- You will lose 3 points for getting this wrong)

Given Name(s): ________________________________
(Please Print)

Student Number: ________________________________

Toronto Email: ________________________________

Signature: ________________________________

You may NOT use calculators, or any electronic devices during the test. You must completely justify your answers. Do NOT remove any pages from the test booklet.

FOR MARKER'S USE ONLY			
Problem 1:	/20	Problem 2:	/15
Problem 3:	/10	Problem 4:	/15
Problem 5:	/25	Problem 6:	/20
Problem 7:	/20	Problem 8:	/19
Problem 9:	/15	Problem 10:	/10
Problem 11:	/21	Problem 12:	/10
		TOTAL:	/200

Problem 1. (20 points, 10 points each).

(i) Let $T : V \to V$ be a linear mapping on finite dimensional space V with eigenvalue λ of multiplicity m. Define a **λ-generalized eigenspace**.

(ii) State the **Cauchy-Schwarz theorem** for inner product spaces.

Problem 2. (15 points). Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 2 \\ 3 & 0 & 0 \end{bmatrix}$ be the matrix of $T : \mathbb{R}^3 \to \mathbb{R}^3$ with respect to the standard basis. Compute $[T]_\alpha^\alpha$ for $\alpha = \{(1, 0, 1), (0, 1, 1), (1, 1, 0)\}$.

Problem 3. (10 points). Let V be a real inner product space. Prove that u and v are orthogonal if and only if $||u|| \leq ||u + av||$ holds for all $a \in \mathbb{R}$.

Problem 4. (15 points). Define the transformation $T : P_2(\mathbb{R}) \to P_2(\mathbb{R})$ via $T(ax^2 + bx + c) = (a-b)x^2 + (b-c)x + (a-c)$. Find a basis for $Im(T)$.

Problem 5. (25 points). Find the canonical form and a canonical basis for

$$N = \begin{bmatrix} 4 & 1 & -1 & 2 \\ -4 & -1 & 2 & -1 \\ 4 & 1 & -1 & 2 \\ -4 & -1 & 1 & -2 \end{bmatrix}$$

Problem 6. (20 points). Let $S \subset V$ be a subspace of finite dimensional vector space V and let $P_S : V \to S$ be an orthogonal projection onto S. Prove that P_S is diagonalizable.

Problem 7. (20 points). Let V be a finite dimensional complex inner product space with Hermitian inner product. Suppose that $T : V \to V$ is linear, Hermitian and has non-negative eigenvalues. Prove that there exists an operator S such that $T = S^*S$. *(The operator S is said to be a square root of T).*

Problem 8. (19 points). Calculate the Jordan canonical form of $A = \begin{bmatrix} 2 & 2 & 4 \\ 1 & 3 & 3 \\ -1 & -2 & -2 \end{bmatrix}$

Problem 9. (15 points). How many different cycle tableaux are there for a nilpotent operator $N : \mathbb{R}^5 \to \mathbb{R}^5$?

Problem 10. (10 points). We define V to be the set of all infinite sequences of real numbers together with the standard scalar addition and multiplication. Namely $V = \{(a_1, a_2, ...) \mid a_j \in \mathbb{R}, j = 1, 2, ...\}$ and, for all $c \in \mathbb{R}$, $c(a_1, a_2, a_3, ...) = (ca_1, ca_2, ca_3, ...)$ and $(a_1, a_2, a_3, ...) + (b_1, b_2, b_3, ...) = (a_1 + b_1, a_2 + b_2, a_3 + b_3, ...)$ holds for all $(a_1, a_2, a_3, ...), (b_1, b_2, b_3, ...) \in V$. Definte $T : V \to V$ via $T(a_1, a_2, a_3, ...) = (a_1, 2!a_2, 3!a_3, ...)$. Is T an isomorphism? If so, prove it. If not, explain why not.

Problem 11. (10 points). Let $p_0, p_1, ..., p_m \in P_m(\mathbb{R})$ be polynomials satisfying $p_j(\pi) = 0$, for $j = 0, ..., n$. Are $p_0, p_1, ..., p_m$ dependent or independent? Prove your answer.

Problem 12. (3 points for each correct answer, minus 2 points for each incorrect answer for a minimum of -14 points or a maximum of 21 points). For the following questions, answer using the word "True" or the word "False". You **don't need to justify your answer** to receive full credit. There's no partial credit.

(i) True/False: If J is in Jordan canonical form and $c \in \mathbb{R}$ then cJ is also in Jordan canonical form.

(ii) True/False: Nilpotent transformations on finite dimensional spaces are diagonalizable.

(iii) True/False: If N is a nilpotent transformation on a finite dimensional space then $\det(N) = 0$.

(iv) True/False: If p_T is the characteristic polynomial of a diagonalizable linear transformation T, then p_T has n distinct roots.

(v) True/False: Let $\mathbb{F}$ be a field. Then the additive identity and multiplicative identity must be unique.

(vi) True/False: Similar matrices are diagonalizable.

(vii) True/False: If $V = \sum_{i=1}^{k} W_i$ and $W_i \cap W_j = \{0\}$ for $i \neq j$, then $V = W_1 \oplus \cdots \oplus W_k$.

(*extra paper*)