MAT 334, Complex Variables

Summer 2020

Course Information

Instructors: Nathan Carruth, ncarruth@math.toronto.edu, LEC 5101.
Matthew Sourisseau, sourisse@math.utoronto.ca, LEC 0101.
Adminstrative questions should be addressed to Nathan at the e-mail above.
(Please note that Nathan does not check or reply to e-mails on Sundays.)

Lecture times: LEC 5101: Tuesday 6 – 7 PM, Thursday 6 – 8 PM
LEC 0101: Wednesday 11 AM – 12 PM, Friday 11 AM – 1 PM.
[NB All times are in Eastern Daylight Time, which is UTC-4.]
Lectures will be live-streamed and you are expected to be in attendance at your scheduled lecture time. Recordings of lectures will not be available except possibly for students in exceptional circumstances such as those in 'Timezone exceptions' below.

Instructor Office Hours: TBA

Course Website: http://www.math.toronto.edu/ncarruth/MAT334_S20/. [This text is not a live link.] All links given in this syllabus are available on the course website. Quercus will also be used for announcements (so please make sure you get e-mails when announcements are posted!) and giving marks, and may also be used for parts of the quizzes/final assessment.

TAs: Stanislav Balchev, Etienne Bilocq, Yucong Jiang, Ren Zhu

Tutorial Times: Please see the listing on the Faculty of Arts and Science's Summer 2020 Timetable, timetable.iit.artsci.utoronto.ca/summer2020.
As with lectures, you are expected to be in attendance at your scheduled tutorial time.

TA Office Hours: TBA (see course webpage for updates).

Textbook: Goursat, Édouard. Functions of a Complex Variable. Hedrick, Earle Raymond and Dunkel, Otto, translators. Ginn and Company: Boston, 1916.
I believe this book is now public domain; scans are available online. I plan to produce a typescript of the sections we cover as the course progresses.

Information specific to this term (please read carefully!)

Expectations about attendance. The lectures and tutorials in this class will be delivered live, and this course should not be thought of as an 'online' course which you can do on your own time but rather as a normal course, temporarily delivered via online methods. Except under special circumstances which require case-by-case exceptions from Nathan, you must be available at the times listed on the timetable for the lecture and tutorial in which you are registered.

We understand that timezone issues in particular may cause difficulties with live participation. On the other hand, there are a range of tutorial and lecture times available, and we expect you to be willing to make an effort to participate live regardless of your timezone. (Nathan has stayed up until almost midnight before to hold office hours and will do so again if needed.) We also understand that circumstances caused by the Covid-19 pandemic may impact your ability to participate live. On the other hand, we hope that the situation has stabilised sufficiently that such circumstances will not suddenly arise mid-term.

If you feel that your situation justifies an exception to this policy, you must contact Nathan immediately, and absolutely no later than the end of the first week of classes (May 8).

Covid-19 situation. While the ongoing Covid-19 pandemic makes all plans contingent to a certain extent, we are confident that major interruptions like those during winter term can be avoided. Please remember that you are a student even if you are not living in Toronto and do not plan unnecessary activities (such as travel) which you would not plan under ordinary circumstances, or which under the present circumstances present a high likelihood of severely compromising your ability to carry out your studies. The pandemic situation by itself is only a valid reason to ask for an accommodation when the circumstances were unforeseeable.

Minimum electronics requirements. Lectures, tutorials, and office hours will be held online either via Blackboard Collaborate or Quercus Conferences (a similar tool), both accessed through Quercus, or Zoom. Quizzes and the final assessment will be downloaded from Quercus or Crowdmark, and solutions must be uploaded through one of those portals. This means that you must have electronic equipment capable of accessing those resources. The University has produced a list of recommended minimum system requirements. Note that for this course, access to (at the very least) a computer with speakers and a camera or scanner is also required. If you have any concerns about this, again, please discuss it with your instructor as soon as possible.

Course outline

The aim of this course is to provide a solid foundation in the basic theory of analytic functions of a complex variable, and a sampling of the applications of this theory. These two aspects may be broken down as follows.

- Theory: The condition of differentiability of a function of a complex variable gives rise to deep and far-reaching consequences. Two main topics we shall discuss are *integral theorems*, which are mainly analytic, and *conformal mapping properties*, which are mainly geometric.
- Applications: We shall show how results arising from the integral theorems allow us to compute definite integrals which are totally intractable by the normal methods of real-variable calculus. We shall also show how the conformal mapping properties of analytic functions allow us to find solutions to certain partial differential equations. Time permitting, we shall discuss the applications of the theory to the study of so-called *special functions*.

We shall cover most of chapters I and II and parts of chapter IV of the text by Goursat.

While the applications to definite integrals are probably the most easily appreciated and widely known applications of the material in this course, we will see that the study of functions of a complex variable is a rich and deep subject with connections to many other parts of mathematics, both pure and applied.

Course goals

By the end of this course, you should understand how differentiability of a function of a complex variable leads to the Cauchy-Riemann equations and the Cauchy integral theorem and integral formula; how these results give rise to Taylor and Laurent series; the significance of branch points and branch cuts; and how knowledge of the complex singularities of a function as encoded in its Laurent series and branch points allows us to choose appropriate contours in the complex plane for the evaluation of definite integrals of real-variable functions. You should also understand the geometric interpretation of the derivative and its role in the theory of conformal maps, and applications of conformal maps to the solution of Laplace's equation in two variables.

More generally, successfully completing this course should give you the knowledge and appreciation of functions of a complex variable necessary for further applications in (for example) ordinary and partial differential equations, special function theory, transform theory, number theory, and so forth.

Prerequisites

The formal prerequisites for this course are multivariable calculus and at least one semester of linear algebra. In particular, we shall assume a strong grasp of partial differentiation and line integrals. Review notes on these topics will be posted to the course website; if you feel at all shaky with either of these after reviewing the notes please talk to one of us sometime during the first week of class.

Classes and Tutorials

Formal attendance will not be taken in classes or tutorials, but you are responsible for the material discussed in both and cannot expect the instructors or TAs to re-teach you the material later. You must write the weekly quiz in your registered tutorial section.

While the lectures and tutorials will be delivered via electronic media, they will be live and this course should not be thought of as an 'online' course which you can do on your own time but rather as a normal course, temporarily delivered via online methods. In particular, you *must* be available at the times listed for lectures and tutorials on the timetable.

The only exception to the foregoing policies is for special circumstances such as those described in 'Timezone exceptions' below. If you feel your situation warrants an exception you must contact Nathan as soon as possible, and no later than the end of the first week of classes (i.e., May 8).

While not strictly required, we highly recommend registering in a tutorial section on the same day as your lecture section; i.e., if you are registered in LEC0101, we highly recommend you enroll in either the Wednesday or the Friday tutorial, while if you are in LEC5101, we highly recommend that you enroll in one of the Tuesday or Thursday tutorials. This will allow for maximum coordination between tutorials and lectures.

Assessment

There will be 10 quizzes, written during tutorials, each worth 8% of the final course mark, and one final assessment, to be written during the final exam period in August, worth 20% of the final assessment. In addition, to account for missed quizzes and reward improvement during the term, the final course mark will be determined by combining the quiz marks with the mark on the final assessment in the following fashion. The final assessment will consist of 10 independent pieces, corresponding to the 10 quizzes, and the mark on the final assessment corresponding to a particular quiz will replace the mark on that quiz if (a) it is higher than the mark on that quiz and (b) the work on the quiz showed a serious attempt to solve the problem. 'Serious effort' will be defined as part of the grading rubric for each quiz, but basically it means that you must have done something more than simply write a list of memorised formulas with no progress towards actually obtaining the answer.

To make the foregoing precise, let Q_i denote the mark on the ith quiz, $i = 1, 2, \ldots, 10$, and let T_i denote the mark on the corresponding ith segment of the final assessment, both as percentages. Define a new quantity F_i as follows: If $T_i > Q_i$ and a serious attempt was shown on quiz i, then $F_i = T_i$; otherwise $F_i = 0.2T_i + 0.8Q_i$. The final mark on the course will be given by

$$F = \frac{1}{10} \sum_{i=1}^{10} F_i.$$

Quizzes and the final assessment will be administered online via either Quercus or Crowdmark. You will not need a printer but you will need a way to produce an electronic copy of your answers (e.g., using a camera or a scanner). For details see 'Minimum electronics requirements' below.

Missed coursework and accommodations

If you miss a quiz or the final assessment for reasons beyond your control, you must contact your instructor as soon as possible, but no later than one week after your situation returns to normal. (Usually this would be when you return to class.) Doctor's notes are no longer required, but in addition to contacting your instructor you must also register your absence using the Absence Declaration Tool on ACORN. If you miss a quiz under such circumstances, you will be deemed to have shown a serious attempt on that quiz and your mark on it will be replaced by your mark on the final assessment. In such cases, if you would like feedback on your progress in the course, you may contact Nathan to request permission to take the quiz informally for feedback only. The quiz will then be written and marked as usual, but the result is solely for feedback purposes and will not under any circumstances be counted towards your mark in the course.

Students who are not able to write the final assessment will be given an opportunity to write a make-up assessment later on in August.

If you are aware of potential absences ahead of time (for example, related to religious observances), please advise your instructor as far in advance as possible.

Marking, returning and remarking of course work

Quizzes will be marked by the TAs working together (not necessarily your section TA). The final assessment will be marked by your instructors, assisted by the TAs as needed.

Quizzes will be given marks and feedback on Quercus or Crowdmark as appropriate. We will aim for quizzes to be marked within one week of writing. Solutions may be provided as considered appropriate, generally no sooner than one week after the quiz.

Remark requests for quizzes should be submitted as soon after the quiz is returned as possible, but must be submitted no more than one week after the quiz. If you feel that an arithmetic error was made in determining your mark, please contact your TA with the relevant information. If you feel that your work was marked unfairly, please contact your TA, indicate that you would like to request a remark, and justify why you think the mark given was incorrect. The quiz will then be remarked. In either case, the new mark (which may be higher or lower than the original) will become the mark for that question or questions.

Please note that "I deserve more marks for this solution" is very rarely a valid reason to ask for a remark. Also, please do not argue with your TAs about your marks. If you believe your TA does not understand your concern, please bring it directly to Nathan's attention.

Cheating. The course policy on this heading can be summarised in three words:

DON'T DO IT!!!

First, some pyrotechnics. We will not mince words: Cheating is a direct insult to your instructor, your TAs and your fellow students. Your instructors and TAs work hard to design, prepare, and deliver course material, provide feedback and guidance, and assess course performance. Your fellow students, who do not cheat, are doing their best to earn the grade they desire by actually putting in the required effort to learn the material. Cheating denigrates all of this and is totally unacceptable.

Cheating in the context of this course includes, but is not limited to, any of the following activities:

- Working with any other individual, inside or outside of the class, on any of the quizzes or the final assessment.
- Posting course material to the internet without the **explicit** permission of your instructor.
- Accessing any online resources of any form while writing a quiz or the final assessment.
- Allowing someone else to write a quiz or the final assessment for you, or doing the same for someone else.
- Submitting any work which is not entirely your own independent work.

All cases of cheating will be taken very seriously and referred to the University for potential disciplinary action. **Disciplinary action for academic misconduct, which includes cheating, can be severe, up to and including expulsion from the University.**

Pyrotechnics hopefully having now had their desired effect, we want to emphasise that our main interest in this class is to help you actually learn the material, not to be police officers. If at any point in the term you are becoming concerned about your performance in the course, please talk to your TA or instructor. We will do what we can to help you learn the course material and get the best mark you can. If you feel that your performance is less than you wanted, the solution is to work hard and ask for help, not to cheat.

Miscellaneous

Please see the course website listed on p. 1 for additional information about the course, including some comments on the choice of textbook.