APM 346, practice problems for week 6.

[The problems below were distilled from homework problems in [1], Chapter 3, by (attempting to!) extract the mathematical core from the physics problem. As I have not yet had time to work all of them out it is possible that I may not have extracted all of the relevant information from the physics, or may have done so incorrectly; if you find any problem totally intractable, please let me know.

Given that these problems come from a textbook I do not intend to post solutions to them. I would however be happy to discuss solutions you have in office hours, or read over a rough write-up.]

1. [See [1], problem 3.2.] Solve Laplace's equation on the ball $r < a$, subject to the boundary condition

$$u_r|_{r=a} = \begin{cases} 0, & \theta < \alpha \\ 1, & \alpha < \theta < \pi \end{cases},$$

where $\alpha \in (0, \pi)$.

2. [See [1], problem 3.4.] Let $n \in \mathbf{Z}$, $n \geq 1$, and solve Laplace's equation on the ball $r < a$ subject to the boundary condition

$$u|_{r=a} = \begin{cases} -1, & \phi \in (\frac{k\pi}{n}, \frac{(2k+1)\pi}{2n}) \\ 1, & \phi \in (\frac{(2k+1)\pi}{2n}, \frac{(k+1)\pi}{n}) \end{cases},$$

where k ranges from 0 to $n-1$. (In other words, the range in ϕ is divided into $2n$ equal pieces, and u on these pieces is required to be alternately -1 or 1.)

3. [This problem requires using the sin/cos basis in the z direction and hence is somewhat outside of the scope of the material we have discussed in class; but if you have a good grasp of what is going on, you should be able to do it – just leave the solutions to Bessel's equation (which in this case are denoted I_m and K_m instead of J_m) as they are: they play the role of the exponential functions (alternatively, sinh/cosh), which means that their values at certain points enter into the results but one doesn't need to calculate any integrals involving them. See [1], 3.10.] Solve Laplace's equation on the cylinder $\{(\rho, \phi, z) | \rho < \rho_0, 0 < z < z_0\}$, subject to the boundary conditions

$$u|_{z=0} = u|_{z=z_0} = 0, \qquad u|_{\rho=\rho_0} = \begin{cases} -1, & 0 < \phi < \pi \\ 1, & \pi < \phi < 2\pi. \end{cases}.$$

References

1. Jackson, J. D. *Classical Electrodynamics*, 3rd ed. Hoboken, NJ: John Wiley & Sons, Inc., 1999.