

APM 346, practice problems for quiz 3.

1. Expand the following functions in terms of the orthogonal basis $\{1, \sin 2n\pi x, \cos 2n\pi x | n \in \mathbf{Z}, n > 0\}$ on the interval $(0, 1)$:

$$f(x) = \begin{cases} 0, & x \in [0, \frac{1}{3}) \\ 1, & x \in (\frac{1}{3}, \frac{2}{3}) \\ 0, & x \in (\frac{2}{3}, 1] \end{cases}$$
$$g(x) = 1 - f(x)$$
$$h(x) = \begin{cases} 1, & x \in [0, \frac{1}{3}) \\ -1, & x \in (\frac{1}{3}, \frac{2}{3}) \\ 0, & x \in (\frac{2}{3}, 1] \end{cases}$$

2. Expand the functions in problem 1 in terms of the basis $\{\sin n\pi x | n \in \mathbf{Z}, n > 0\}$ on the interval $(0, 1)$.
3. The same as 2, except use the basis $\{1, \cos n\pi x | n \in \mathbf{Z}, n > 0\}$.
4. Make up your own step functions on $(0, 1)$ and repeat the above three problems on them.