We give a brief review of integration of piecewise-defined functions. Suppose that we have a function $F : [a, b] \to \mathbf{R}$ (or $\mathbf{C}$ would work just as well), suppose that $a_0 = a < a_1 < \cdots < a_n = b$ is some finite sequence of numbers in $[a, b]$, let $F_k : [a_{k-1}, a_k] \to \mathbf{R}$ ($k = 1, 2, \ldots, n$) be continuous[1], and suppose that for each $k = 1, 2, \ldots, n$ we have

$$F(x) = F_k(x) \quad \text{for all}\, x \in [a_{k-1}, a_k],$$

in other words, that we have the piecewise definition

$$F(x) = \begin{cases} F_1(x), & x \in [a, a_1] \\ F_2(x), & x \in [a_1, a_2] \\ & \vdots \\ F_n(x), & x \in [a_{n-1}, b] \end{cases}$$

Then it can be shewn that F is integrable on $[a, b]$, and

$$\int_a^b F(x)\, dx = \sum_{k=1}^{n} \int_{a_{k-1}}^{a_k} F_k(x)\, dx.$$

If some F_k is zero, then its integral over its domain $[a_{k-1}, a_k]$ will also be zero, and hence it will not contribute to the sum and may be dropped.

The foregoing applies in particular when we are computing the coefficients in the expansion of a function (say $f(x)$) on $[a, b]$ in terms of a complete orthogonal set of functions on $[a, b]$; see, for example, the solution to problem 2 on homework 3. In particular, since coefficients in such an expansion do not depend on x, and must therefore take into account the function values over the entire interval, in cases where f has a piecewise definition it will be necessary to use a formula like that above to calculate inner products involving f.

[1] 'Integrable' would work just as well here.