Summary:

- Any integrable function can be expanded in a series of complete orthogonal functions, and the coefficient of the function e_i is simply the inner product $\frac{(f,e_i)}{(e_i,e_i)}$, where the inner product is given by $(f,g) = \int_a^b f(x)\overline{g(x)}\,dx$.
- On the interval $[0,1]$, two complete orthogonal sets are $\{1, \cos 2k\pi x, \sin 2k\pi x | k \in \mathbf{Z}, k > 0\}$.
- This allows us to determine the *Fourier series* of a function f by computing the inner products $(f,1)$, $(f, \cos 2k\pi x)$, $(f, \sin 2k\pi x)$, as well as the lengths $(1,1)$, etc.

More Later