APM 346 (Summer 2019), Homework 9.

APM 346, Homework 9. Due Monday, July 22, at 8.00 AM EDT. To be marked completed/not completed.

1. Using the eigenfunctions and eigenvalues for the Laplacian on the cylinder $C = \{(\rho, \phi, z) | \rho < 1, 0 \le z \le 1\}$ derived in class, solve the following problem on C:

$$\nabla^2 u = z \begin{cases} 0, & \rho < \frac{1}{2} \\ \rho^3 \cos 3\phi, & \frac{1}{2} < \rho < 1 \end{cases}, \quad u|_{\partial C} = 0.$$

2. Using the eigenfunctions and eigenvalues for the Laplacian on the unit ball $B = \{(r, \theta, \phi) | r < 1\}$ derived in class, solve the following problem on B:

$$\nabla^2 u = 3\sin^2\theta \cos 2\phi \begin{cases} r^2, & r < \frac{1}{2} \\ 0, & \frac{1}{2} < r < 1 \end{cases}, \quad u|_{\partial B} = 0.$$

3. Solve the following problem on the unit cube Q:

$$\nabla^2 u = 0, \quad u|_{x=0} = u|_{x=1} = u|_{y=0} = u|_{y=1} = 0, \quad u|_{z=0} = \sin \pi x \sin 2\pi y, \quad u|_{z=1} = 0.$$

4. Recall the function χ defined in problem 1 of assignment 8:

$$\chi(x) = \begin{cases} 0, & 0 \le x < \frac{1}{2} \\ 1, & \frac{1}{2} < x \le 1 \end{cases}.$$

Let u_0 denote the solution to problem 3. Solve the following problem on the unit cube Q:

$$\frac{\partial u}{\partial t} = \nabla^2 u, \quad u|_{\partial Q} = u_0|_{\partial Q}, \quad u|_{t=0} = \chi(x)\chi(y)\chi(z).$$

[Optional: compute the coefficients in the series for u for two choices of ℓ, m, and n, one small (say $\ell = m = n = 1$) and another large (say $\ell, m, n > 10$). Compare the ratio of these coefficients for $t = 0$ and $t = 10$.]

Does the function u have a limit as $t \to +\infty$?