

APM 346, Homework 8. Due Monday, July 15, at 6.00 AM EDT. To be marked completed/not completed.

Using our derivation of the eigenfunctions and eigenvalues of the Laplacian in class, solve the following problems.

1. Write out a series expansion for the solution to the following problem on $Q = \{(x, y, z) | 0 \leq x, y, z \leq 1\}$:

$$\nabla^2 u = \chi(x)\chi(y)\chi(z), \quad u|_{\partial Q} = 0,$$

where ∂Q is the boundary of the cube Q and

$$\chi(x) = \begin{cases} 0, & 0 \leq x < \frac{1}{2} \\ 1, & \frac{1}{2} < x \leq 1 \end{cases}.$$

2. Write out a series expansion for the solution to the following problem on $Q \times (0, +\infty)$, where Q is as in problem 1:

$$\frac{\partial u}{\partial t} = \nabla^2 u, \quad u|_{\partial Q} = 0, \quad u|_{t=0} = \sin \pi x \sin \pi y,$$

where we denote an arbitrary point in $Q \times (0, +\infty)$ by (x, y, z, t) .