

APM 346 (Summer 2019), Homework 5.

APM 346, Homework 5. Due Monday, June 10, at 6.00 AM EDT. To be marked completed/not completed.

1. Solve the following boundary-value problem on the region $\{(r, \theta, \phi) | 1 < r < 2\}$:

$$\nabla^2 u = 0, \quad u|_{r=2} = \begin{cases} 1, & 0 \leq \theta < \frac{\pi}{2} \\ -1, & \frac{\pi}{2} < \theta \leq \pi \end{cases}, \quad u_r|_{r=1} = \begin{cases} 0, & 0 \leq \theta < \frac{\pi}{2} \\ 1, & \frac{\pi}{2} < \theta \leq \pi \end{cases}.$$

[Hint: use Legendre polynomial identities to calculate $\int_0^1 P_\ell(x) dx$ and $\int_{-1}^0 P_\ell(x) dx$.]

2. Solve the following boundary-value problem on the region $\{(r, \theta, \phi) | r < 2\}$:

$$\nabla^2 u = 0, \quad u|_{r=2} = x(1 + y).$$

(Here $x = r \sin \theta \cos \phi$ and $y = r \sin \theta \sin \phi$ are the standard Cartesian coordinates corresponding to the given spherical coordinate system.)