APM 346 (Summer 2019), Homework 3.

APM 346, Homework 3. Due Monday, May 27, at 6.05 AM EDT. To be marked completed/not completed.

1. Recall the following boundary-value problem on the interval [0, 1] from Homework 2:

$$f'' = -\lambda^2 f, \qquad f(0) = 0, \qquad f'(1) = -f(1).$$

Show that if $(\lambda_1, f_1)$ and $(\lambda_2, f_2)$ are two solutions to this boundary-value problem, $\lambda_1, \lambda_2 > 0$, $\lambda_1 \neq \lambda_2$, then $f_1$ and $f_2$ are orthogonal with respect to the standard inner product $(f, g) = \int_0^1 f(x)\overline{g(x)}\,dx$. (You may use the solution posted on the course website, or work directly from the equation and boundary conditions above.)

2. Solve the following boundary-value problem on $[0, 1] \times [0, 1]$:

$$\nabla^2 u = 0, \qquad f(x,0) = \begin{cases} 1, & x \in [0, \frac{1}{2}) \\ 0, & x \in (\frac{1}{2}, 1] \end{cases}, \qquad f(x,1) = \begin{cases} 0, & x \in [0, \frac{1}{2}) \\ 1, & x \in (\frac{1}{2}, 1] \end{cases},$$
$$f(0, y) = 0, \qquad f(1, y) = 0.$$

(You may use the expansion of $f(x, 0)$ given in the lecture notes.)

3. (a) Write $x^4$ on $(-1, 1)$ as a series of Legendre polynomials. (Hint: the series has only finitely many terms. But you need to prove this!)

(b) (Optional) Is the series expansion from (a) valid outside of the interval $(-1, 1)$? Is this likely to matter for our applications of Legendre polynomials?