APM 346, Homework 2. Due Monday, May 20, at 6 AM EDT. To be marked completed/not completed.

1. Use the identity $e^{3i\theta} = \left(e^{i\theta}\right)^3$ ($\theta \in \mathbf{R}$) to find an expression for $\cos 3\theta$ in terms of $\cos\theta$ and $\sin\theta$.

2. Find all numbers $\lambda > 0$ for which there is a nonzero function f on $(0,1)$ satisfying

$$f'' = -\lambda^2 f, \qquad f(0) = 0, \qquad f'(1) = -f(1).$$

Also find the corresponding functions f. (Note: it is enough to find an equation which λ must satisfy. It is in general not possible to solve this equation.)

3. (You need only do one of problems 3 and 4.) Suppose that $A_n \in \mathbf{R}$, $n = 0, 1, 2, \ldots$, $B_n \in \mathbf{R}$, $n = 1, 2, \ldots$, are such that

$$x = \frac{1}{2}A_0 + \sum_{n=1}^{\infty} \left(A_n \cos 2n\pi x + B_n \sin 2n\pi x\right)$$

for $x \in (0,1)$. Find an expression for the A_n and B_n.

4. (You need only do one of problems 3 and 4.) Suppose that $A_n \in \mathbf{C}$, $n = 0, 1, 2, \ldots$, are such that

$$x = \sum_{n=0}^{\infty} A_n e^{2in\pi x}$$

for $x \in (0,1)$. Find an expression for the A_n.