APM 346 (Summer 2019), Homework 12.

APM 346, Homework 12. Due Monday, August 12, at 11.59 PM EDT. To be marked turned in/not turned in.

1. Using the eigenfunctions and eigenvalues for the Laplacian on the unit disk $D = \{(\rho, \phi) | \rho < 1\}$ derived in class, solve the following problem on $(0, +\infty) \times D$:

$$\frac{\partial u}{\partial t} = \nabla^2 u, \quad u|_{\partial D} = 0, \quad u|_{t=0} = \rho^4 \cos 4\phi.$$

2. Using Fourier transforms in space, solve the following problem on $(0, +\infty) \times \mathbf{R}^1$:

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \quad u|_{t=0} = e^{-x^2}, \quad u_t|_{t=0} = 0.$$

3. Again using Fourier transforms, consider the following problem on $(0, +\infty) \times \mathbf{R}^1$; give an integral expression for the solution, and evaluate it as far as possible:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + u, \quad u|_{t=0} = e^{-x^2}.$$

What is the behaviour of this solution in the limit $t \to +\infty$?