APM 346, Homework 10. Due Monday, July 29, at 6.00 AM EDT. To be marked completed/not completed.

1. Starting from separation of variables, give the series expansion to the solution for the following problem in terms of an appropriate set of eigenfunctions of the Laplacian on the unit cube $Q = \{(x, y, z)|0 \leq x, y, z \leq 1\}$:

$$\nabla^2 u = \begin{cases} 1, & 0 \leq z < \frac{1}{2} \\ -1, & \frac{1}{2} < z \leq 1 \end{cases}, \quad \partial_\nu u|_{\partial Q} = 0, \quad u(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}) = 0,$$

where ∂_ν denotes the outward normal derivative on the surface (e.g., on the surface $\partial Q \cap \{z = 0\}$, it is $-\frac{\partial}{\partial z}$).

2. Compute the Fourier transforms of the following functions:

$$f(x) = \begin{cases} 1, & x \in [-1, 1] \\ 0, & \text{otherwise} \end{cases}.$$

$$f(x) = \begin{cases} 1 - |x|, & x \in [-1, 1] \\ 0, & \text{otherwise} \end{cases}.$$

$$f(r, \theta, \phi) = \begin{cases} 1, & r \leq 1 \\ 0, & \text{otherwise} \end{cases}.$$

$$f(x) = e^{-ax^2}, \quad a \in \mathbf{R}, a > 0.$$

$$f(r, \theta, \phi) = e^{-ar^2}, \quad a \in \mathbf{R}, a > 0.$$

$$f(x) = xe^{-ax^2}, \quad a \in \mathbf{R}, a > 0.$$

[For the fifth of these, it may be simpler to change to rectangular coordinates.]