APM 346 (Summer 2019), Homework 1.

APM 346, Homework 1. Due Monday, May 13, at 6 AM EDT. To be marked completed/not completed.

1. Calculate the indicated derivatives.

(a) $\frac{d}{dx}\left(10x^6 - 5x^3 + 4x^2 - 7x + 1\right)$.

(b) $\frac{d}{dx}\left(\ln\left[5x^2 - 3x + 100\right]\right)$.

(c) $\frac{d}{dx}\left(e^{5x^{10} - 10x^5 + 102}\right)$.

(d) $\frac{d}{dx}(\sin 2x)$.

(e) $\frac{d}{dx}(\cos kx)$, k a constant.

(f) $\frac{\partial}{\partial y}(\cos k_1 x \, \sin k_2 y)$, k_1, k_2 constants.

(g) $\frac{\partial}{\partial z}\left(\sin^{-1}\left(\ln\left(\cos\left(\tan\left(xyz + x^2 + 10xy - 100\right)\right)\right)\right)\right)$.

2. Evaluate the following expressions.

(a) $\nabla\left(x^2 + y^2\right)$.

(b) $\nabla\left(x^2 + y^2 - 2z^2\right)$.

(c) div $(x\mathbf{i} + y\mathbf{j} + 10\mathbf{k})$.

(d) div $\left(\nabla\left(x^2 + y^2 - 2z^2\right)\right)$.

(e) div $(\nabla(e^y \sin x))$.

3. Evaluate the following integrals. (You must show your work to get credit.)

(a) $\int_0^{2\pi} x^2 \sin x \, dx$.

(b) $\int_0^{2\pi} x \sin(kx) \, dx$, k a constant.

(c) $\int_0^{+\infty} x e^{-x} \, dx$.

(d) $\int_0^{2\pi} e^x \cos x \, dx$.

(e) $\int_0^{2\pi} \sin k_1 x \sin k_2 x \, dx$, $k_1, k_2 \in \mathbf{Z}$, $k_1 \neq k_2$.

(f) Same as (e), but with $k_1 = k_2$.

(g) $\int_0^{2\pi} \sin k_1 x \cos k_2 x \, dx$, k_1, k_2 any two integers.

4. Evaluate the following integrals.

(a) $\iint_R \sin x \, \sin y \, dA$, $R = [0, \pi] \times [0, \pi]$.

(b) $\iint_R e^{-\left(x^2 + y^2\right)} \, dA$, R the unit disk in the xy-plane.

(c) $\iiint_R \sin\left(x^2 + y^2 + z^2\right)^{\frac{3}{2}} \, dV$, R the unit ball in xyz-space.

5. Consider the two-dimensional vector space of functions on the interval $[0, 1]$

$$V = \{a \sin \pi x + b \cos \pi x | a, b \in \mathbf{R}\}.$$

Let $B = \{\sin \pi x, \cos \pi x\} \subset V$.

(a) Prove that B is a basis for V. (Hint: Wronskian!)

(b) Find the matrix representation $[T]_B$ of the operator T in the basis B, for (i) $T = \frac{d}{dx}$; (ii) $T = \frac{d^2}{dx^2}$.

6. Consider the differential equation $\frac{d^2 y}{dx^2} = -4y$.

(a) Find the set of all solutions to this equation.

(b) Find a basis for this solution set. (You must prove that your answer is in fact a basis.)

(c) (Optional) Can you find the set of all solutions to $\frac{d^2 y}{dx^2} + 4y = \sin 4x$?

7. Find all (a) local and (b) global maxima of $f(x, y) = e^y \cos x$ on the rectangle $[0, 2\pi] \times [0, 1]$.