

University of Toronto
FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATIONS, JUNE 2001

Year 1 – Programs 1, 2, 3, 4, 6, 7, 8, 9

MAT 187H1F
CALCULUS II
Examiner: D. Burbulla

INSTRUCTIONS

Non-programmable calculator permitted; no other aids allowed.

Present your solutions to all of the following questions in the exam booklets supplied.
The marks for each question are indicated in parentheses.

TOTAL MARKS: 100.

1. (15 marks: 5 marks each) Find the following:

(a) $\int x e^x dx$

(b) the length of the curve with parametric equations

$$x = \cos t; y = \sin t; z = t^{3/2}$$

for $0 \leq t \leq 1$.

(c) a unit tangent vector to the curve $\mathbf{r} = \sin^{-1}t \mathbf{i} + \ln(t + 1) \mathbf{j}$ at the point for which $t = 0$.

2. (15 marks) Find the general solution to each of the following differential equations:

(a) (5 marks) $\frac{dy}{dx} = 3y + 5$

(b) (10 marks) $\frac{dy}{dx} + \frac{xy}{x^2 + 1} = \frac{1}{x}$

3. (15 marks: 5 marks each) The following three parts are *not* related.

(a) Find the 5th degree Taylor polynomial of $f(x) = \frac{x}{1-x}$ at $x = 0$

(b) Find the interval of convergence of the power series $f(x) = \sum_{n=0}^{\infty} \frac{n}{n^2+1} x^n$

(c) Approximate the value of $\int_0^{1/3} \frac{\sin x}{x} dx$ correct to within 0.0001, and explain why your approximation is correct to within 0.0001

4. (15 marks) Consider the cardioid with polar equation $r = 1 - \sin \theta$.

(a) (5 marks) Plot the cardioid, and label all x and y intercepts.

(b) (5 marks) Find the length of the cardioid.

(c) (5 marks) Find the area of the region within the cardioid.

5. (10 marks) Find the critical points of $f(x, y) = 4x^3 - 6xy^2 + 3y^4$ and at each critical point determine whether f has a relative maximum point, a relative minimum point, or a saddle point.

6. (10 marks) Do the following infinite series converge or diverge? Justify your answer.

(a) (3 marks) $\sum_{n=1}^{\infty} \frac{n^2 + n - 1}{n^{5/2} - n^{3/2} + 4}$

(b) (3 marks) $\sum_{n=1}^{\infty} \frac{\sin(1/n)}{n}$

(c) (4 marks) $\sum_{n=1}^{\infty} \frac{2^n \ln n}{5^n \sqrt{n}}$

7. (10 marks) Let $f(x) = \frac{x^2}{(1+x^2)^2}$.

(a) (6 marks) Use the binomial series expansion for $(1+x^2)^{-2}$ to find the Maclaurin series for $f(x)$ and its radius of convergence.

(b) (4 marks) What is the exact value of $\sum_{n=1}^{\infty} \frac{n(-1)^n}{3^{2n}}$?

8. (10 marks) Find $\int_1^{\infty} \frac{x+1}{x^2+x^4} dx$.