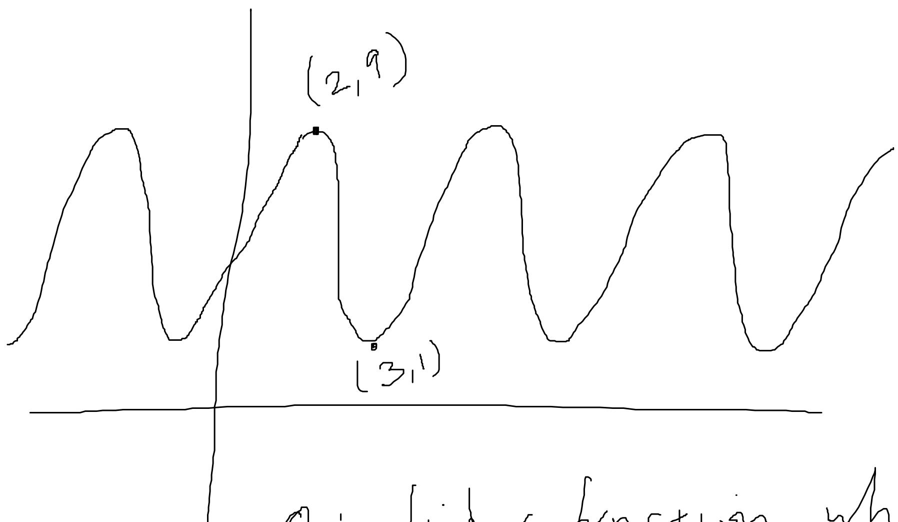
Here's another way to do the problem I discussed in class



Q: find a function whose graph is the above.

1) Know it's sin or cosine. Let's pick sin
The amplitude is $9-1=8$. So the function will involve $4\sin(\dots)$. But $4\sin(\dots)$ ranges from 4 to -4 our graph ranges from 1 to 9. So it's $4\sin(\dots)+5$

2) We need to figure out what goes inside the sin. To do this, we'll need to use the given points $(2,9)$ and $(3,1)$.

$$9 = 4\sin\left(a(\theta-2)+\pi/2\right)+5$$ where we will use the second point $(3,1)$ to find a. Why do we know the thing in the box works? plug in $\theta=2$ and check

Now we use the second point $(3,1)$ to find a.

$$1 = 4\sin\left(a(3-2) + \pi/2\right) + 5$$

Since $\theta = 3$ is the location of the first minimum after $\theta = 2$, we know that

$$a(3-2) + \frac{\pi}{2} = 3\frac{\pi}{2}$$

$$\Rightarrow a \cdot 1 = \pi \quad \Rightarrow \quad a = \pi$$

$$f(\theta) = 4\sin\left(\pi(\theta-2) + \frac{\pi}{2}\right) + 5 \text{ works!}$$

<u>Note 1</u>: In the second step, I could have used the point $(2,9)$ as follows:

$$9 = 4\sin\left(a(\theta-2) + 5\pi/2\right) + 5$$

in which case when I used the second point $(3,1)$ I would have

$$1 = 4\sin\left(a(3-2) + 5\pi/2\right) + 5$$

$$\Rightarrow a(3-2) + 5\pi/2 = 7\pi/2$$

$$\Rightarrow a = \pi$$

and so $f(\theta) = 4\sin\left(\pi(\theta-2) + \frac{5\pi}{2}\right) + 5$ also works.

Note that

$$4\sin\left(\pi(\theta-2)+\frac{5\pi}{2}\right)+5$$

$$= 4\sin\left(\pi(\theta-2)+\frac{\pi}{2}+2\pi\right)+5$$

$$= 4\left[\sin\left(\pi(\theta-2)+\pi/2\right)\cos(2\pi) + \cos\left(\pi(\theta-2)+\pi/2\right)\sin(2\pi)\right]+5$$

$$= 4\sin\left(\pi(\theta-2)+\pi/2\right)+5.$$

So the answers we got describe the same function.

Note 2: What if we'd used cos instead of sin?

by the same logic as before,

$4\cos(\ldots)+5$ is a good start,

we need to find what goes into the cos. Using the point $(2,9)$,

$$9 = 4\cos(a(\theta-2))+5$$

Using the point $(3,1)$

$$1 = 4\cos(a(3-2))+5$$

$\Rightarrow a(3-2) = \pi$ (since $\theta=3$ is the first minimum after $\theta=2$, we use π.)

$\Rightarrow \quad a = \pi \quad$ as before.

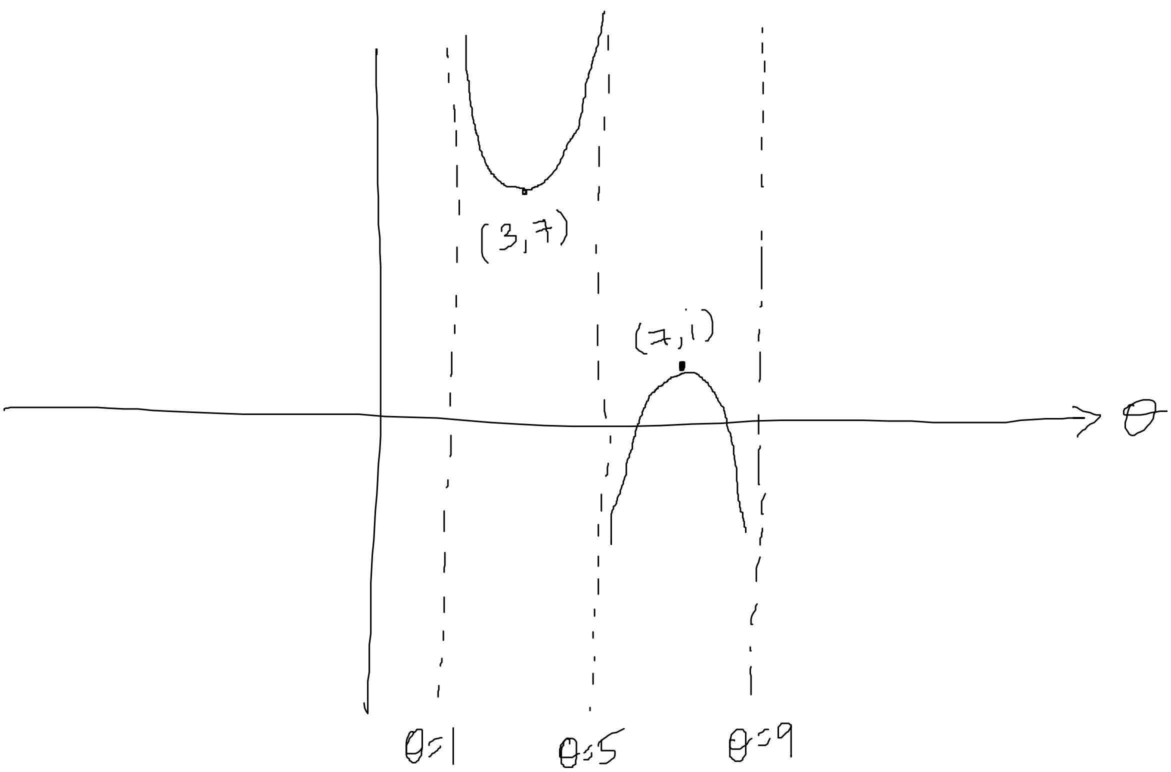
so $\quad 4\cos(\pi(\theta-2)) + 5$ also works!

Note that

$$4\cos(\pi(\theta-2)) + 5 = 4\sin(\pi(\theta-2) + \pi/2) + 5$$

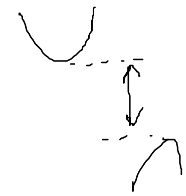
so again we have the same function.

Find the function whose graph is



we'll use sec or csc. to build the function.

$\sec(\theta)$ has range $(-\infty,-1] \cup [1,\infty)$. That is the distance



is 2. In our graph, that distance is $7-1=6$ So we want $3\sec$ to get the right distance.

$3\sec$ has range $(-\infty,-3] \cup [3,\infty)$ while the graph is of a function with range $(-\infty,1] \cup [7,\infty)$. So we need to raise the function by 4.

$$\Rightarrow \quad 3\sec(\ldots) + 4.$$

Now to find what goes inside the sec.

$$7 = 3\sec(a(\theta-3)) + 4$$ (used the point $(3,7)$)

Use $(7,1)$ to find a:

$$1 = 3\sec(a(7-3)) + 4.$$

Since $(7,1)$ is the location of the first local maximum to the right of $(3,7)$, we know

$$a(7-3) = \pi$$

$$a(7-3) = \pi$$

$$\Rightarrow 4a = \pi$$

$$\Rightarrow a = \pi/4$$

So the graph is the graph of the function

$$f(\theta) = 3\sec\left(\frac{\pi}{4}(\theta-3)\right) + 4.$$

As before, we could have used csc instead or could have introduced a phase shift.