Mat 135 Sept 29, 2004

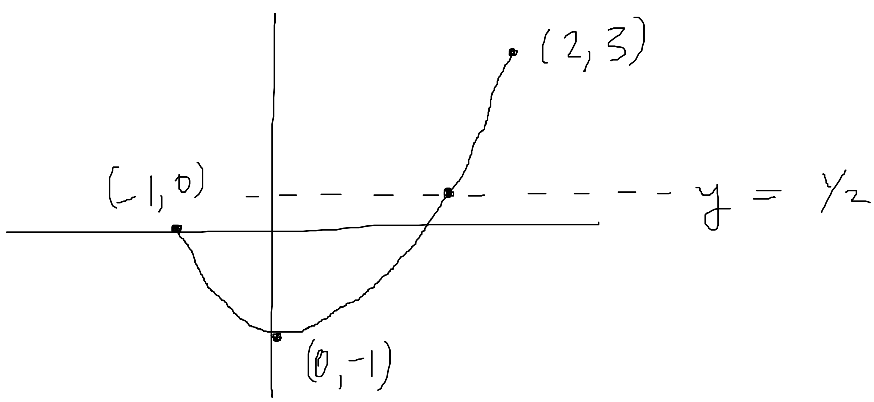
The Intermediate Value Theorem: Suppose that f is continuous on the closed interval $[a\ b]$ and $f(a) \neq f(b)$

let N be any number between $f(a)$ and $f(b)$.

Then there exists a number c in (a, b) such that $f(c) = N$.

ex: Let $[a, b] = [-1, 2]$ and $f(x) = x^2 - 1$

if $N = 1/2$ then



$c^2 - 1 = 1/2 \Rightarrow c^2 = 3/2 \Rightarrow c = \sqrt{3/2}$ which is in the interval $(a, b) = (-1, 2)$ ✓

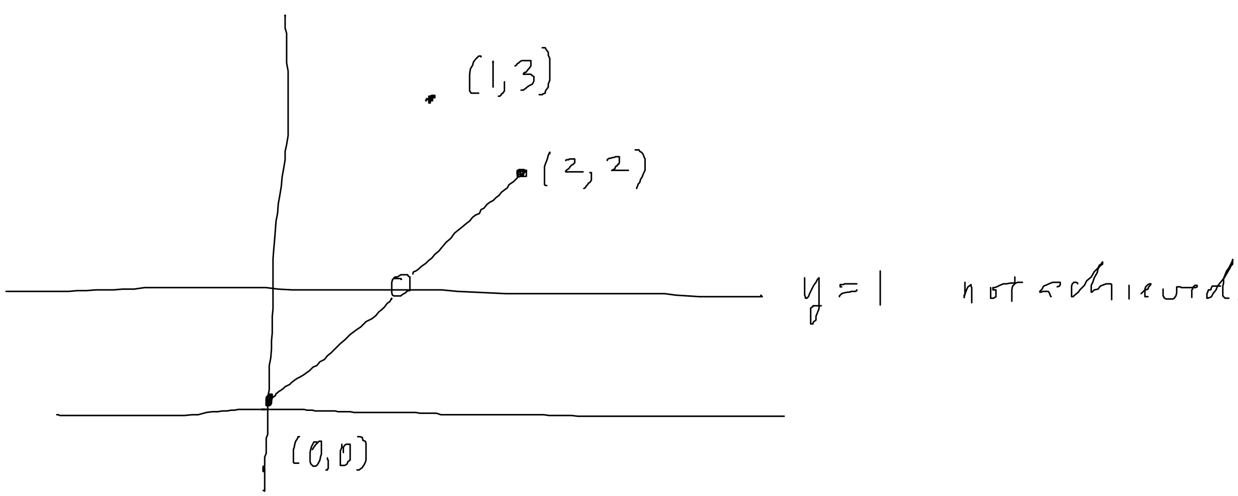
Why do we require the function be continuous?

Consider $f(x) = \begin{cases} \dfrac{x^2-2x+1}{x-1}+1 & x \neq 1 \\ 3 & x = 1 \end{cases}$

on the interval $[a,b] = [0,2]$. Then $f(0) = 0$ and $f(2) = 2$ and

$N = 1$ is between $f(0)$ and $f(2)$.

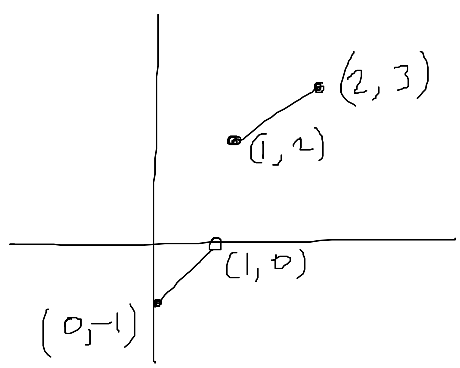
But there is no c in $(0,2)$ such that $f(c) = 1$



if $f(x) = \begin{cases} x+1 & x \geq 1 \\ x-1 & x < 1 \end{cases}$

and $[a,b] = [0,2]$ then $f(0) = -1$ and $f(2) = 3$.

Q: What are some values between $f(0)$ and $f(1)$ that are not achieved

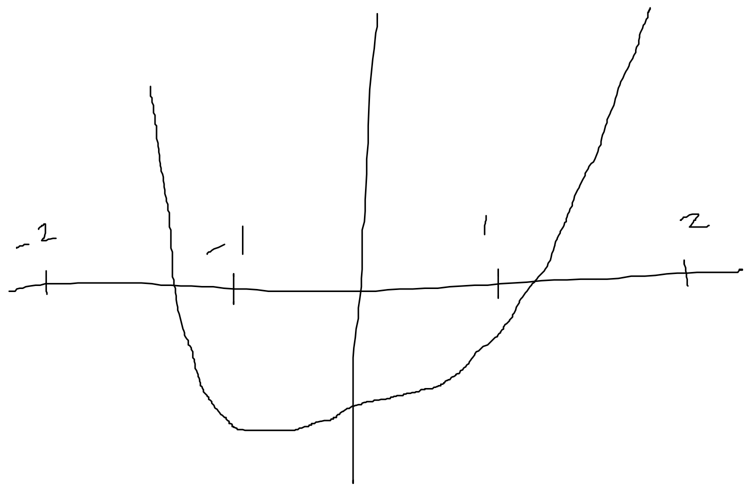


A: there are no c between 0 and 2 such that $0 \leq f(c) < 2$.

#47. Show that there is a point c in $(1,2)$ such that $f(c) = 0$ when $f(x) = x^4 + x - 3$

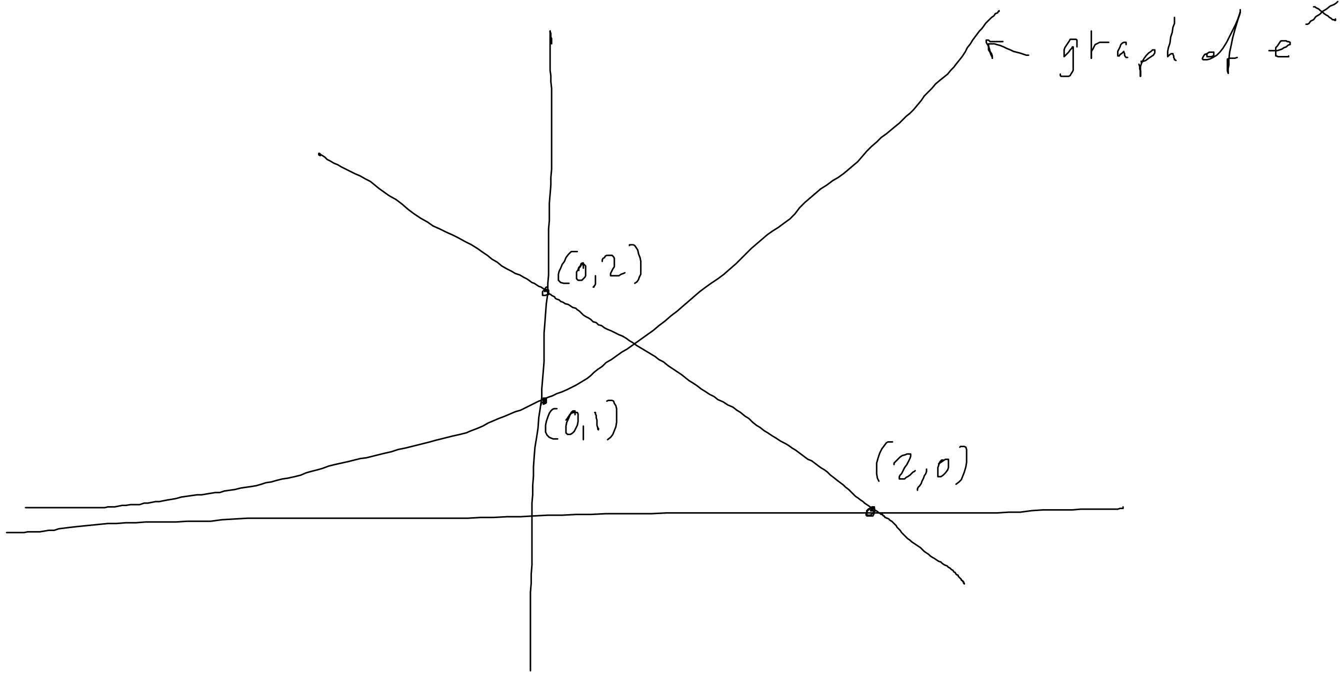
$f(1) = -1 \quad f(2) = 15$

the graph of f:



so we could use the intermediate value theorem to show there's a root in $(-2,-1)$ too!

#51 prove the equation $e^x = 2-x$ has at least one real root



the graph of $2-x$ is above the graph of e^x at $x=0$ and is below the graph of e^x at $x=+2$

Let $f(x) = e^x - 2 + x$ then

$f(0) = -1$ $f(2) = 2$

so given any N between -1 and 2 there is a c in $(0,2)$ such that $f(c) = N$.

Specifically, there's a c such that $f(c) = 0$, as desired.