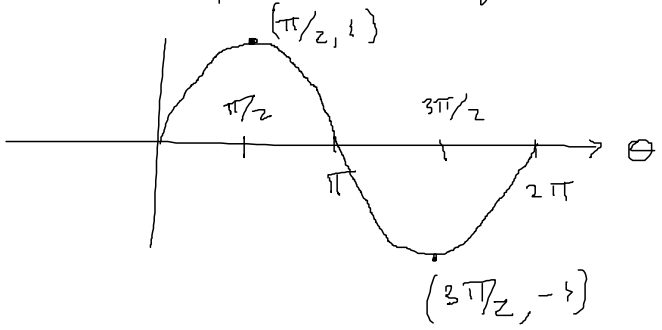
Sept 15, 2004

Want to graph trig functions.

Start by focussing on sin



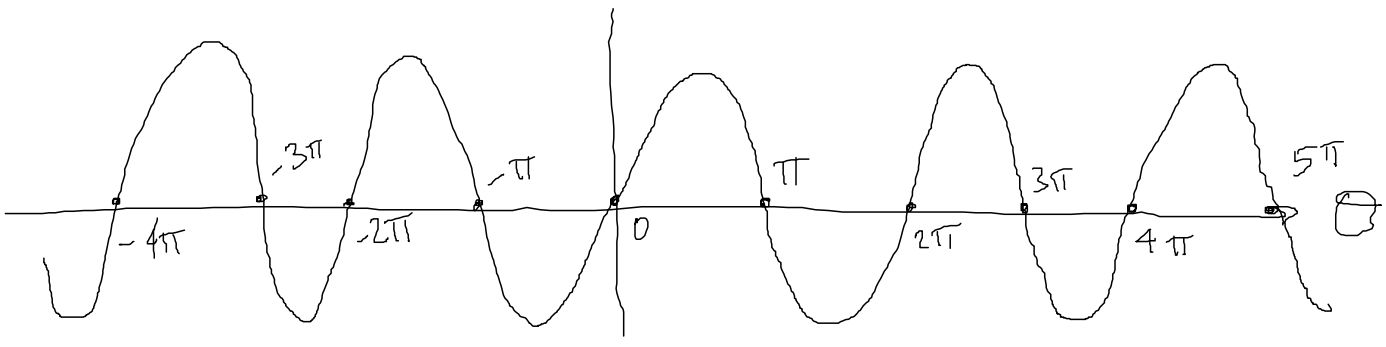
we know that $\sin(\theta) = \frac{y}{r}$ where $r = \sqrt{x^2+y^2}$

and so $|\sin(\theta)| = \left|\frac{y}{r}\right| = \frac{|y|}{r} \leq 1$ for all θ.

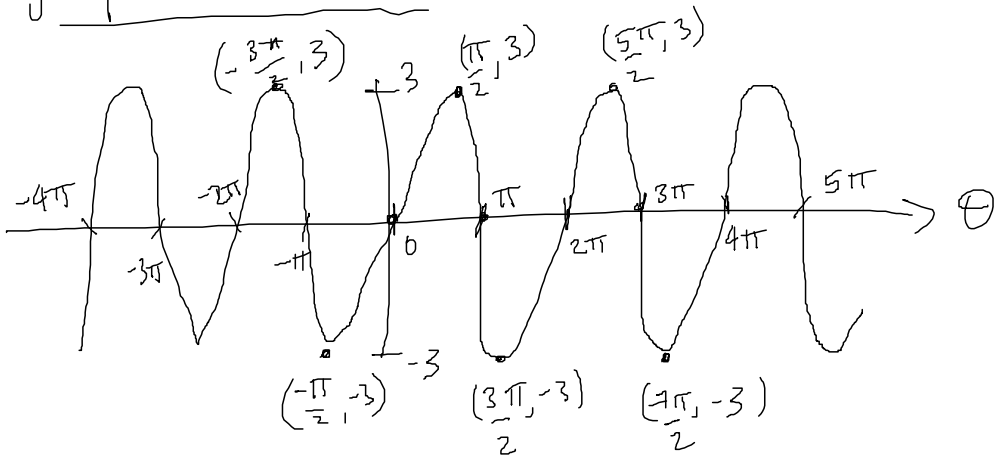
Also, using the periodicity of sin, we get the rest of the graph:

$$\sin(2\pi+\theta) = \sin(2\pi)\cos(\theta) + \cos(2\pi)\sin(\theta)$$
$$= \sin(\theta)$$

$$\sin(k2\pi+\theta) = \sin(k2\pi)\cos(\theta) + \cos(k2\pi)\sin(\theta)$$
$$= \sin(\theta) \quad \text{for any integer } k.$$



graph $3\sin(\theta)$



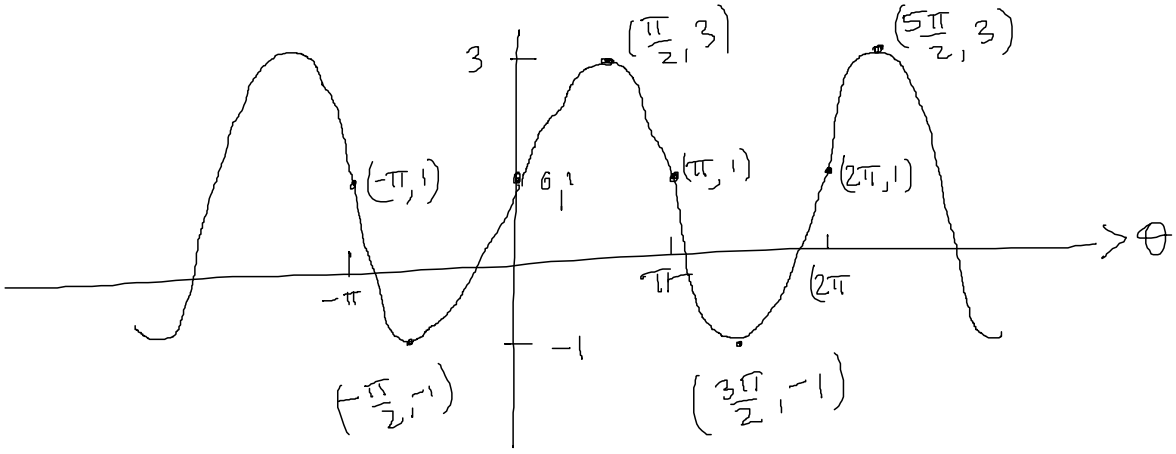
Make sure to put enough "tick marks" on the axes so that the reader can tell that you know what the period and amplitude are. Also, make sure to label some points on the graph.

Graph $2\sin(\theta) + 1$

You know that $|\sin(\theta)| \leq 1$ for all θ

So $-2 \leq 2\sin\theta \leq -2$ for all θ

So $-1 \leq 2\sin\theta + 1 \leq -3$ for all θ



graph $\sin(3\theta)$

We know that $|\sin(3\theta)| \le 1$ for all θ.

find some key points to plot...

$\sin(3\theta) = 0$ if $3\theta = \ldots -2\pi, -\pi, 0, \pi, 2\pi, \ldots$

if $\theta = \ldots -\frac{2}{3}\pi, -\frac{\pi}{3}, 0, \frac{\pi}{3}, \frac{2\pi}{3}, \ldots$

$\sin(3\theta) = 1$ if $3\theta = \ldots -\frac{3\pi}{2}, \frac{\pi}{2}, \frac{5\pi}{2}, \ldots$

if $\theta = \ldots -\frac{3\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}, \ldots$

$\sin(3\theta) = -1$ if $3\theta = \ldots -\frac{5\pi}{2}, -\frac{\pi}{2}, \frac{3\pi}{2}, \ldots$

if $\theta = \ldots -\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{3\pi}{6}, \ldots$

That is

$\sin(3\theta) = 0$ if $\theta = 0 + k \cdot \frac{\pi}{3}$ for an integer k

$\sin(3\theta) = 1$ if $\theta = \frac{\pi}{6} + k \cdot \frac{2\pi}{3}$ for an integer k

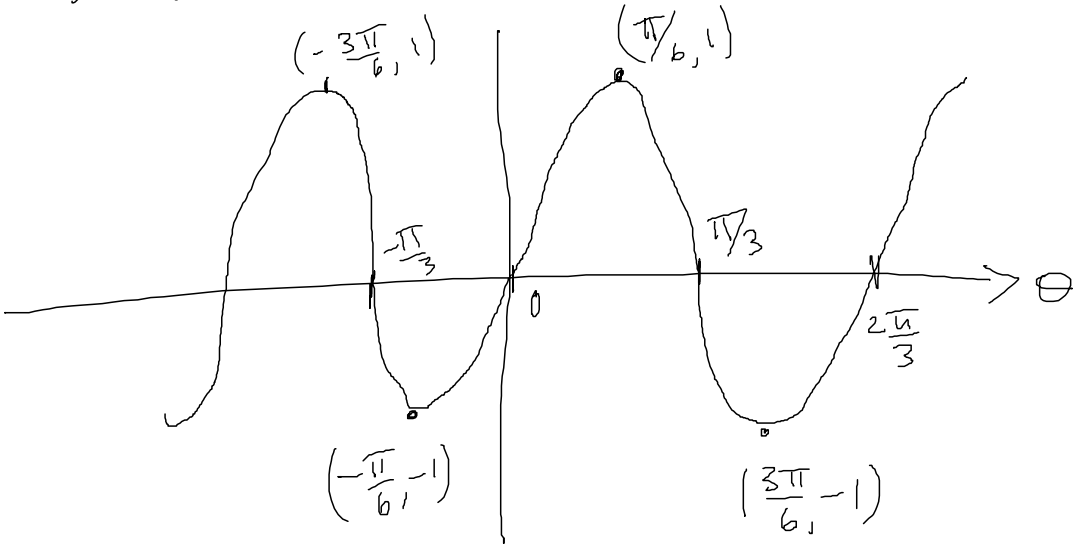
$\sin(3\theta) = -1$ if $\theta = \frac{3\pi}{6} + k \cdot \frac{2\pi}{3}$ for an integer k

The period of $\sin(3\theta)$ is $\frac{2\pi}{3}$. Why?

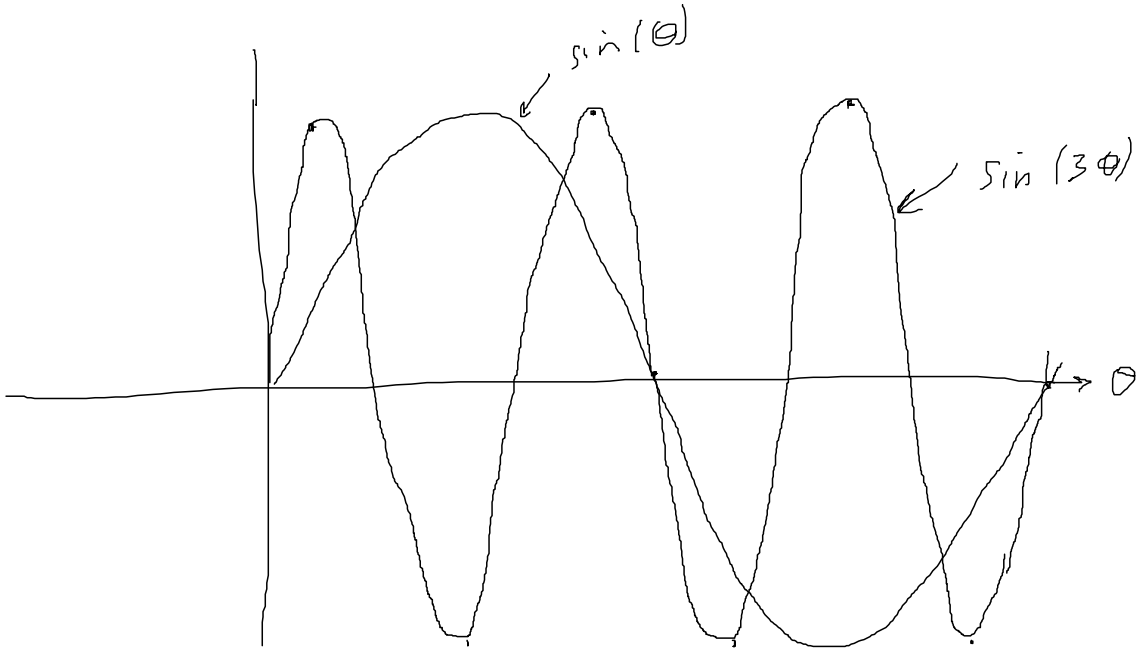
$\sin\left(3\left(\theta + \frac{2\pi}{3}\right)\right) = \sin(3\theta + 2\pi) = \sin(3\theta)$. So

$f(\theta) = \sin(3\theta)$ has $f\left(\theta + \frac{2\pi}{3}\right) = f(\theta)$ for all θ. (This shows the period is $\frac{2\pi}{3}$.

We now have enough information to plot $\sin(3\theta)$

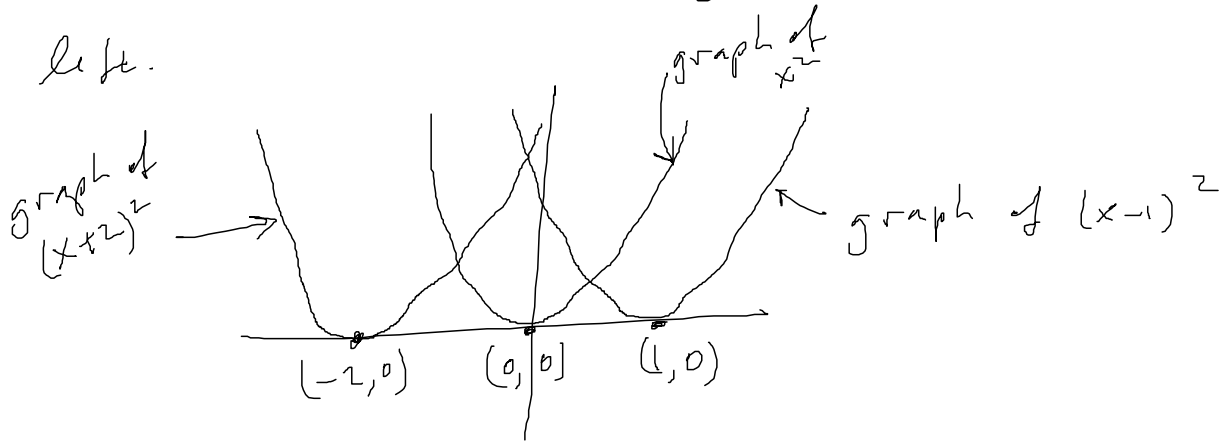


Note: except for the labelling, this looks exactly the same as the graph of $\sin(\theta)$! The way to see the shorter period is to plot two graphs at once:

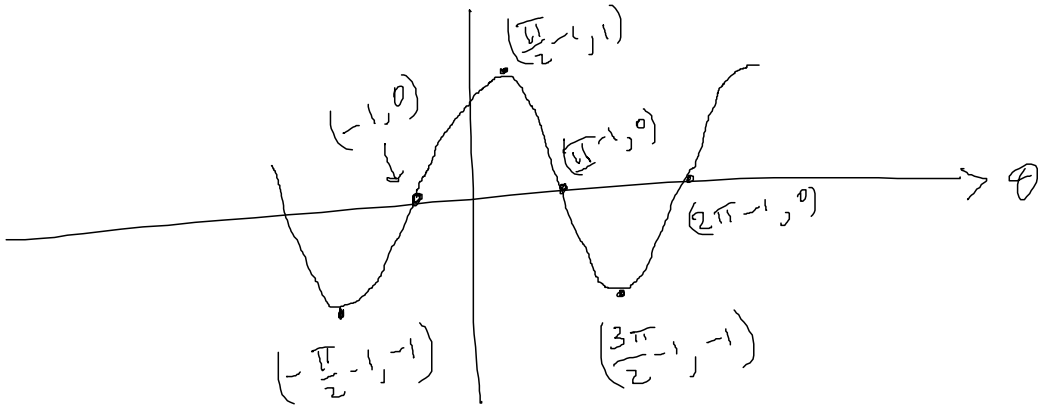


plot $\sin(\theta+1)$

you know how to plot shifts of graphs already: if $f(x) = x^2$ then $f(x-1) = (x-1)^2$ shifts the graph 1 unit to the right and $f(x+2) = (x+2)^2$ shifts the graph 2 units to the left.



So the graph of $\sin(\theta+1)$ is the graph of $\sin(\theta)$ shifted to the left by 1:



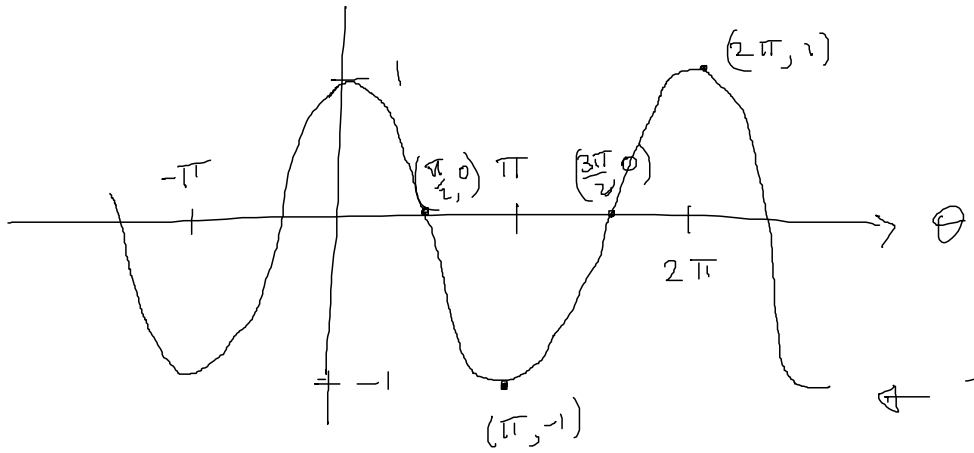
Similarly, the graph of $\sin(3\theta-4) = \sin(3(\theta-4/3))$ is the graph of $\sin(3\theta)$ shifted to the right by $4/3$.

plot $\sin(\theta + \pi/2)$

this is the graph of $\sin\theta$, shifted to the left by $\pi/2$. On the other hand,

$$\sin(\theta + \pi/2) = \sin(\theta)\cos(\pi/2) + \cos(\theta)\sin(\pi/2)$$
$$= \cos(\theta).$$

So we see that it's the graph of $\cos(\theta)$.



← two for the price of one! ☺

$\sin(\theta + \pi/2)$ and $\cos(\theta)$

Okay, now that you can plot $\cos(\theta)$, make sure that you can plot:

$\cos(2\theta)$, $3\cos(\theta)$, $-\cos(\theta)$, and

$4\cos(2\theta - 1) + 8$

Now to plot $\tan(\theta)$.

$\tan(\theta) = \dfrac{\sin(\theta)}{\cos(\theta)}$ $\Rightarrow$ undefined wherever $\cos(\theta) = 0$.

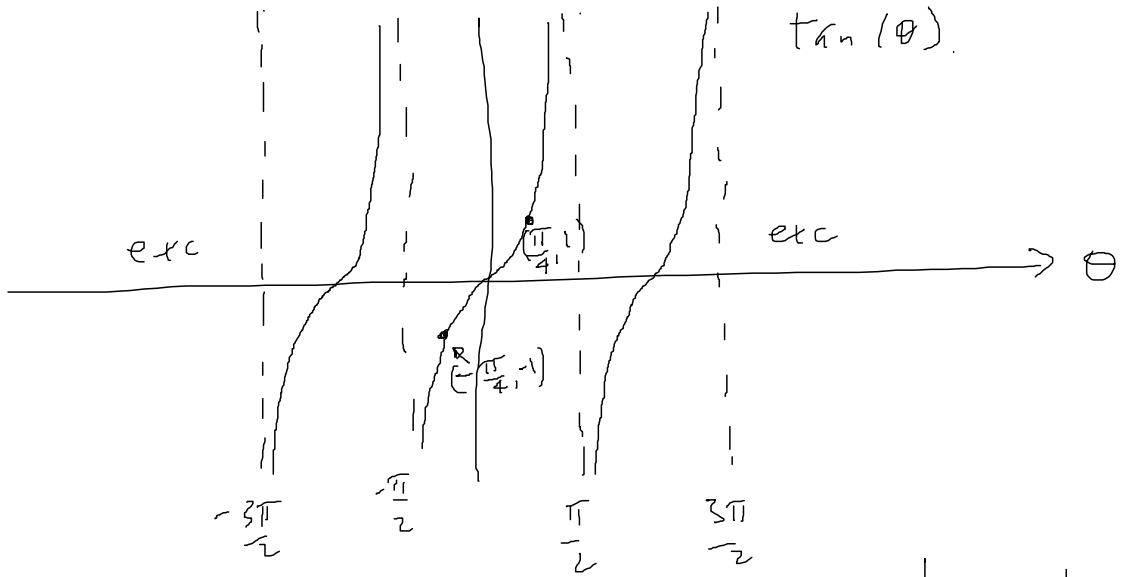
$\Rightarrow$ $\tan(\theta)$ not defined at $\ldots -3\pi/2, -\pi/2, \pi/2, 3\pi/2, \ldots$

$\tan(\theta) > 0$ for $0 < \theta < \pi/2$

$\tan(\theta) < 0$ for $-\pi/2 < \theta < 0$

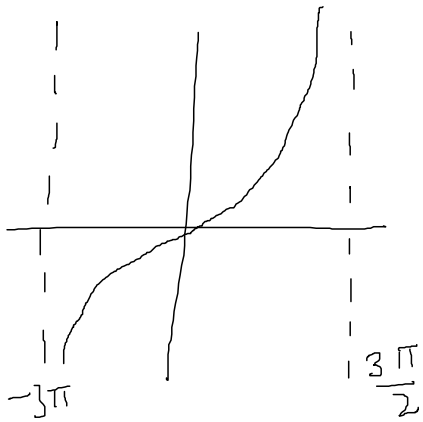
as $\theta \to \pi/2$, $\tan(\theta) \to \infty$

as $\theta \to -\pi/2$, $\tan(\theta) \to -\infty$



tan(2θ)

$\tan\left(\frac{\theta}{3}\right)$



see book for graphs of

$\cot(\theta)$, $\csc(\theta)$, $\sec(\theta)$.

Make sure you can plot

$3\tan(2\theta) + 4$

$-\sec(\theta) - 3$

$\tan(2\theta - 1)$

$\csc(3\theta + 2)$

$7\sec(\theta/4 - 9)$

and so on.

And do the problems in the book !!!