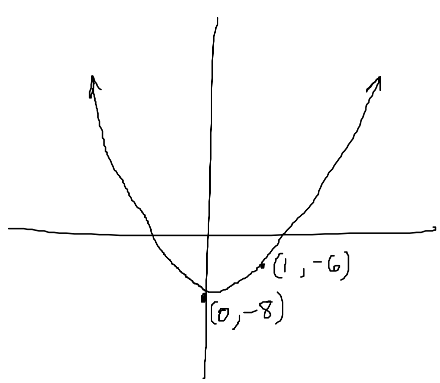
Mat 135 Nov 8, 2004

§ 4.1 Maximum and minimum values

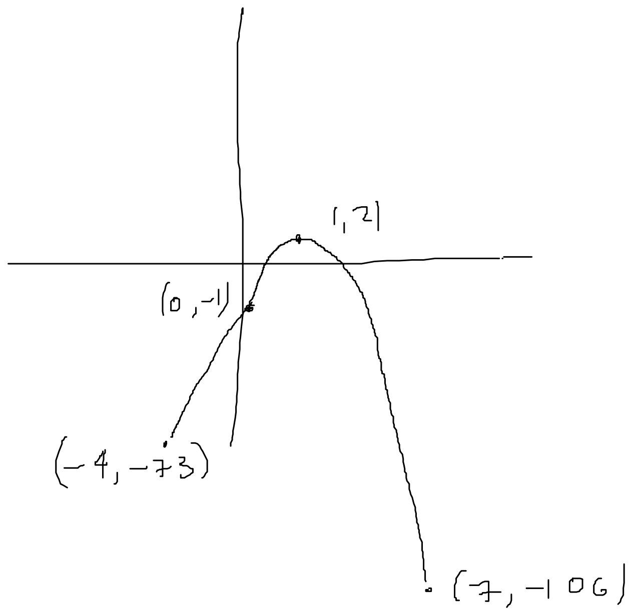
defn: a function f has an absolute maximum (or global maximum) at c if $f(c) \geq f(x)$ for all x in the domain of f. The number $f(c)$ is called the maximum value of f on D (= the domain). Similarly, f has an absolute minimum at c if $f(c) \leq f(x)$ for all x in the domain of f. and the number $f(c)$ is called the minimum value of f on D. The maximum and minimum values of f are called the extreme values of f.

ex: $f(x) = 2x^4 - 8$ on $(-\infty, \infty)$



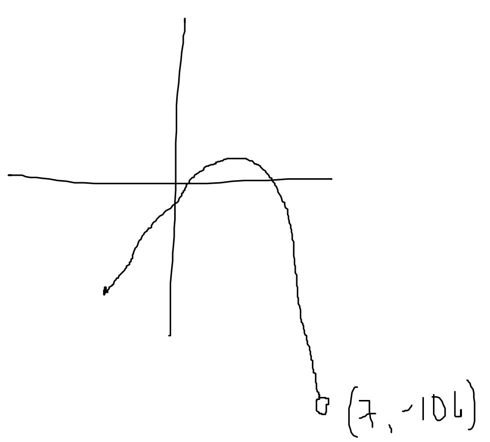
f has absolute minimum at 0. Minimum value is -8

ex: $f(x) = -3(x-1)^2 + 2$ on $[-4, 7]$



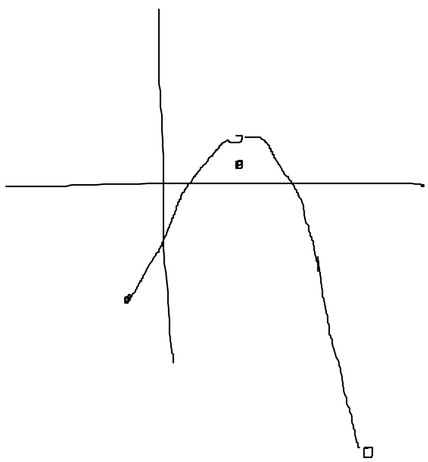
f has an absolute maximum at 1. Maximum value is 2
f has an absolute minimum at 7. Minimum value is -106

ex: $f(x) = -3(x-1)^2 + 2$ on $[-4, 7)$ ← note: 7 not included



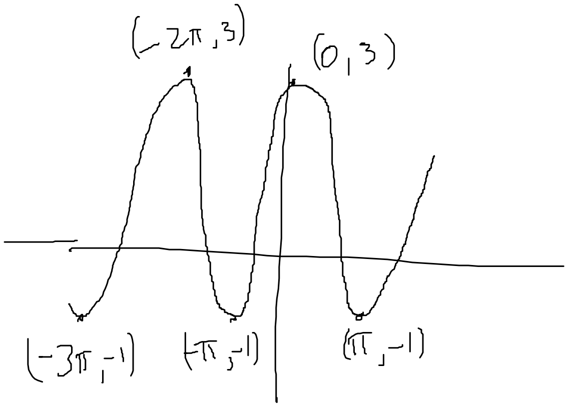
f has an absolute maximum at 1. f has no absolute minimum on $[-4, 7)$

ex: $f(x) = \begin{cases} -3(x-1)^2+2 & -4 \le x < 1 \text{ or } 1 < x < 7 \\ 1 & x = 1 \end{cases}$



f has no absolute maximum on $[-4,7)$. f has no absolute minimum on $[-4,7)$

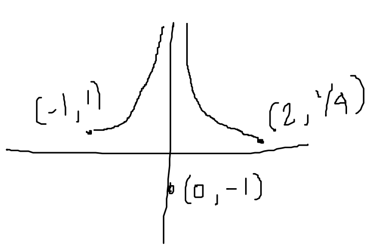
ex: $f(x) = 2\cos(x) + 1$ on $[-10, 5)$



f has an absolute maximum at -2π and at 0. The maximum value is 3.

f has an absolute minimum at -3π, $-\pi$, and at π. The minimum value is -1.

ex: $f(x) = \begin{cases} \frac{1}{x^2} & -1 \le x < 0 \text{ and } 0 < x \le 2 \\ -1 & x = 0 \end{cases}$



f has an absolute minimum at 0. The minimum value is -1.

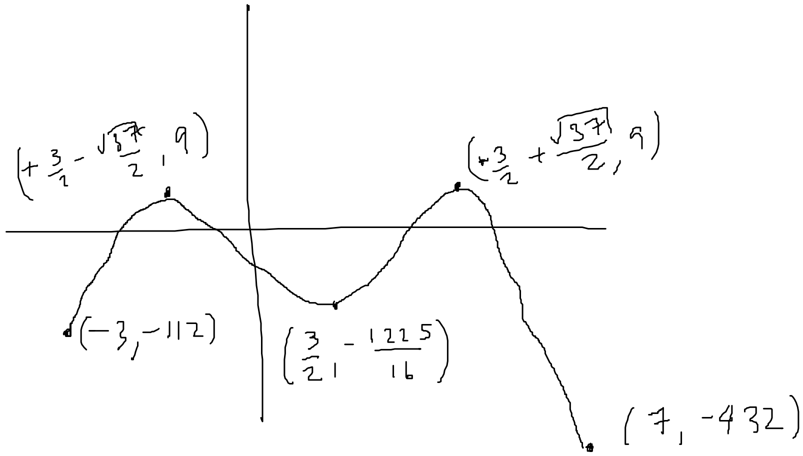
f has no absolute maximum.

<u>defn</u>: A function f has a <u>local</u> <u>maximum</u> (or <u>relative</u> <u>maximum</u>) at c if $f(x) \le f(c)$ when x is near c. (This means that $f(c) \ge f(x)$ for all x in some open interval containing c.) Similarly, f has a <u>local</u> <u>minimum</u> at c if $f(c) \le f(x)$ when x is near c.

<u>NOTE</u>: From the definition, local maxima and local minima cannot occur at endpoints. If the domain of f is $[a, b]$ then neither a nor b can be local maxima or local minima. (This is odd, since they could be absolute maxima or absolute minima. But that's the way Stewart wrote the book. I would have written the book differently and would allow endpoints to be local maxima/local minima. But so be it.)

<u>Note</u>: If you're really curious about <u>why</u> the definition implies that local maxima and local minima can't occur at endpoints, email me or come to my office hours.

ex: $f(x) = -(x+2)(x+1)(x-4)(x-5)$ on $[-3, 7]$



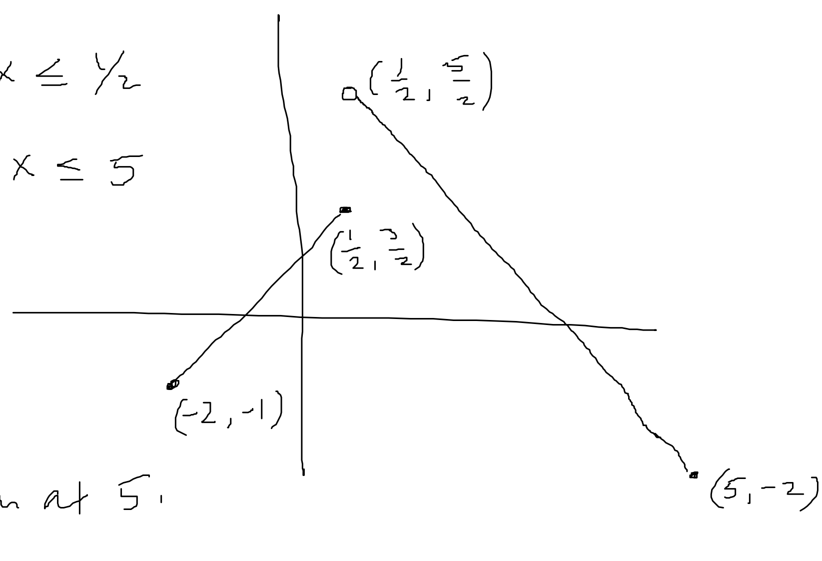
f has absolute maxima at $\frac{3}{2} \pm \frac{\sqrt{37}}{2}$

f has local maxima at $3/2 \pm \sqrt{37}/2$

f has an absolute minimum at 7

f has a local minimum at $3/2$

ex: $f(x) = \begin{cases} x+1 & -2 \le x \le 1/2 \\ -x+3 & 1/2 < x \le 5 \end{cases}$



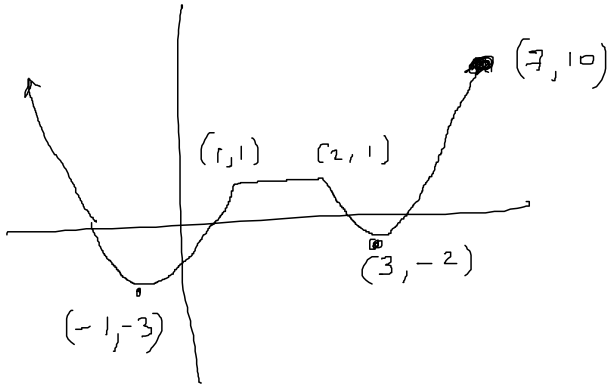
f has an absolute minimum at 5.

f has no local minima

f has no absolute maximum

f has no local maxima. (You have hopes for $1/2$ but while it's true that $f(1/2) \ge f(x)$ for x slightly to the left of $1/2$, it's not true for x slightly to the right of $1/2$.)

ex:



f is defined on $(-\infty, 7]$.

abs max: none
 local max: every point in [1, 2]

abs min: −1
 local min: −1 and 3 and every point in (1, 2)