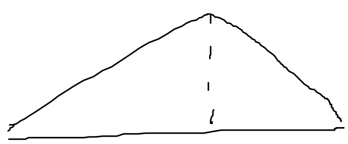
Mat 135 Nov 5, 2004 § 3.10 Related rates

\# 15 The altitude of a triangle is increasing at the rate of 1 cm/min while the area of the triangle is increasing at the rate of 2 cm^2/min. At what rate is the base of the triangle changing when the altitude is 10 cm and the area is 100 cm^2



$h(t)$ = height at time t

$b(t)$ = base at time t

$$A(t) = \frac{1}{2} h(t)\, b(t)$$

$$\Rightarrow \quad \frac{dA}{dt} = \frac{1}{2}\frac{dh}{dt}\, b + \frac{1}{2} h(t) \frac{db}{dt}$$

We've been asked for $\frac{db}{dt}$.

$$\frac{db}{dt} = \frac{\frac{dA}{dt} - \frac{1}{2} b(t) \frac{dh}{dt}}{\frac{1}{2} h(t)}$$

We've been given $\frac{dA}{dt} = 2 \quad \frac{dh}{dt} = 1$

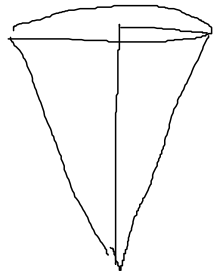
and at the time in question

$h(t_c) = 10 \quad A(t_c) = 100.$

We need $b(t_c)$. But $b(t_c) = \frac{2A(t_c)}{h(t_c)} = \frac{(2)(100)}{10} = 20$

$$\frac{db}{dt}(t_c) = \frac{2 - \frac{1}{2}\cdot 20 \cdot 1}{\frac{1}{2}(10)} = -\frac{8}{5} \text{ cm/min.}$$

\# 19 Water is leaking out of an inverted conical tank at a rate of 10,000 cm^3/min at the same time that water is being pumped into the tank at a constant rate. The tank has height 6 m and the diameter at the top is 4 m. If the water level is rising at the rate of 20 cm/min when the height of the water is 2 m, find the rate at which water is being pumped into the tank.

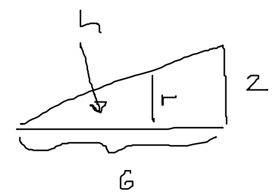


volume $= \frac{1}{3}\pi r^2 h$

by similar triangles,

$$\frac{r}{h} = \frac{2}{6}$$

$$\Rightarrow h = 3r$$



diameter
$= 2 \cdot$ radius

$V(t) =$ volume at time $t = \frac{1}{3}\pi (r(t))^2 h(t)$ where
$h(t) =$ height of water at time t and
$r(t) =$ radius of the water surface at time t.

$$\frac{dV}{dt} = \frac{2}{3}\pi r(t)\frac{dr}{dt}h(t) + \frac{1}{3}\pi r(t)^2\frac{dh}{dt}$$

We're told that at the critical time

$$h(t_c) = 2m \Rightarrow r(t_c) = \frac{2}{3} m$$

We're also told that $\frac{dh}{dt}(t_c) = 20 \text{ cm/min}$

Since $h = 3r$, it follows that

$$\frac{dr}{dt}(t_c) = \frac{1}{3}\frac{dh}{dt}(t_c) = \frac{20}{3} \frac{cm}{min}$$

So $$\frac{dV}{dt}(t_c) = \frac{2}{3}\pi\left(\frac{2}{3}\right)\left(\frac{20}{3}\right)2 + \frac{1}{3}\pi\left(\frac{2}{3}\right)^2(20) \ \frac{m^2 \cdot cm}{min}$$

$$= \frac{80}{9}\pi \times (100)^2 \ \frac{cm^3}{min}$$

$$= \frac{800,000}{9}\pi \ \frac{cm^3}{min}$$ = rate of H_2O being added − rate of leakage

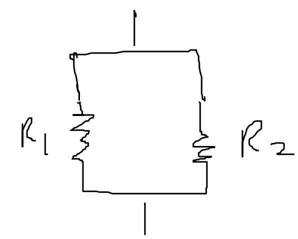
$\Rightarrow$ rate H_2O being added

$$= \text{rate of leakage} + \frac{800,000}{9}\pi \ \frac{cm^3}{min}$$

$$= \left(10,000 + \frac{800,000}{9}\pi\right)\frac{cm^3}{min} \cong 2.89 \times 10^5$$

#29 If two resistors w/ resistances R_1 and R_2 are connected in parallel then the total resistance R, measured in ohms (Ω), is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$



If R_1 and R_2 are increasing at rates $.3\ \Omega/sec$ and $.2\ \Omega/sec$, how fast is R changing when $R_1 = 80\ \Omega$ and $R_2 = 100\ \Omega$?

$$R = \frac{R_1 R_2}{R_1 + R_2}$$

$$\Rightarrow \frac{dR}{dt} = \frac{\frac{dR_1}{dt}\left(R_2(t)\right)^2 + R_1(t)^2 \frac{dR_2}{dt}}{\left(R_1(t) + R_2(t)\right)^2}$$

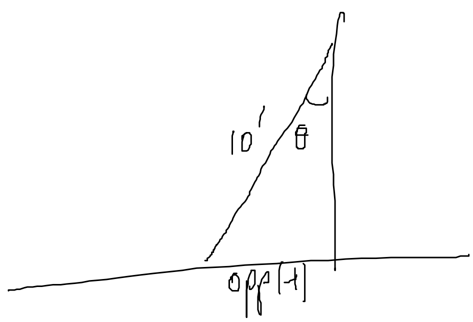
at the time in question,

$$R_1(t_0) = 80\ \Omega \quad R_2(t_0) = 100\ \Omega$$

$$\frac{dR_1}{dt}(t_0) = .3\ \frac{\Omega}{sec} \quad \frac{dR_2}{dt}(t_0) = .2\ \frac{\Omega}{sec}$$

$$\rightarrow \frac{dR}{dt}(t_0) = \frac{(.3)(100)^2 + (80)^2(.2)}{(80+100)^2} = \frac{107}{810}\ \Omega/sec \cong 0.132\ \frac{\Omega}{sec}$$

\# 31 A ladder 10 feet long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a speed of 2 feet/sec, how fast is the angle between the top of the ladder and the wall changing when the angle is $\frac{\pi}{4}$ rad?



$$\sin(\theta(t)) = \frac{opp(t)}{10}$$

$$\Rightarrow \cos(\theta(t))\frac{d\theta}{dt} = \frac{1}{10}\frac{d}{dt}opp(t)$$

at the critical time, $\theta(t_c) = \pi/4$

we're told that $\frac{d}{dt}opp(t) = \frac{2\text{ feet}}{\text{sec}}$

$$\Rightarrow \frac{d\theta}{dt}(t_c) = \frac{1}{10\text{ feet}}\left(\frac{2\text{ feet}}{\text{sec}}\right)\frac{1}{\cos(\pi/4)} = \frac{1}{5}\,\frac{2}{\sqrt{2}}\,\frac{1}{\text{sec}}$$