§ 4.10 Antiderivatives.

defn: A function F is called an antiderivative of f on an interval I if $F'(x) = f(x)$ for all x in I.

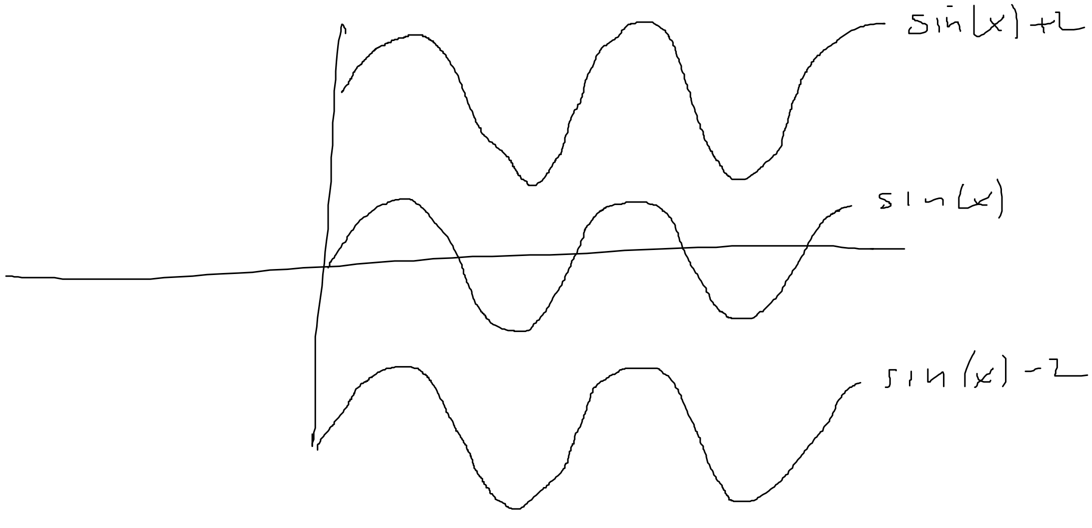
ex: if $f(x) = \cos(x)$ then

$F(x) = \sin(x)$

$F(x) = \sin(x) + 2$

$F(x) = \sin(x) - 2$

are all antiderivatives of f on $(-\infty, \infty)$.



all three graphs have the exact same tangent line slopes ⇒ have the same derivative: $\cos(x)$.

ex.

#3 find the most general antiderivative of $f(x) = 1 - x^3 + 5x^5 - 3x^7$

$$F(x) = x - \frac{x^4}{4} + \frac{5}{6}x^6 - \frac{3}{8}x^8 + C_1$$

where C_1 is a constant.

#11 find the most general antiderivative of

$$f(u) = \frac{u^4 + 3\sqrt{u}}{u^2}$$

$$= u^2 + 3u^{-3/2}$$

$$\Rightarrow F(u) = \frac{1}{3}u^3 - 6u^{-1/2} + C_1$$

#15 $f(x) = 2x + \dfrac{5}{\sqrt{1-x^2}}$

$$\Rightarrow F(x) = x^2 + 5\sin^{-1}(x) + C_1$$

#17 find the antiderivative that satisfies given condition

$$f(x) = 5x^4 - 2x^5 \qquad F(0) = 4$$

$$F(x) = x^5 - \frac{2}{6}x^6 + C_1$$

$$F(0) = C_1 = 4 \Rightarrow F(x) = x^5 - \frac{1}{3}x^6 + 4$$

#24 $f'''(t) = t - \sqrt{t}$ find f.

$$f''(t) = \frac{t^2}{2} - \frac{2}{3}t^{3/2} + C$$

$$f'(t) = \frac{t^3}{6} - \frac{2}{3}\cdot\frac{2}{5}t^{5/2} + Ct + D$$

$$f(t) = \frac{t^4}{24} - \frac{2}{3}\cdot\frac{2}{5}\cdot\frac{2}{7}t^{7/2} + \frac{Ct^2}{2} + Dt + E$$

$$F(t) = \frac{t^4}{24} - \frac{8}{105}t^{7/2} + \frac{Ct^2}{2} + Dt + E$$

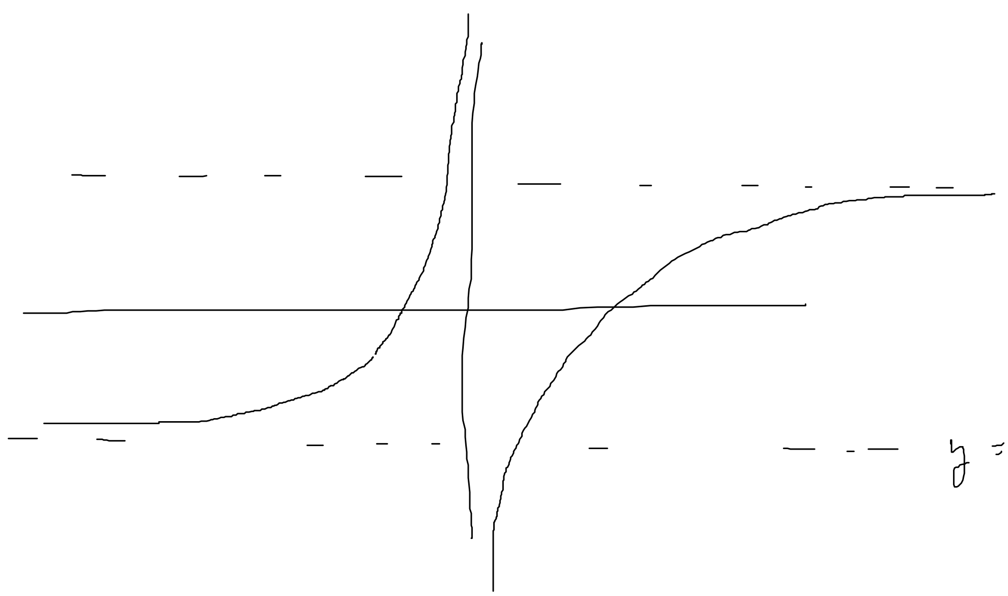
#30 $f'(x) = \frac{3}{x^2}$ $\qquad f(1) = f(-1) = 0$

tricky!
$$f(x) = \begin{cases} \frac{-3}{x} + a & x > 0 \\ \frac{-3}{x} + b & x < 0 \end{cases}$$

$f(1) = \frac{-3}{1} + a = 0 \Rightarrow a = 3$

$f(-1) = \frac{-3}{-1} + b = 0 \Rightarrow b = -3$

$$\Rightarrow f(x) = \begin{cases} -\frac{3}{x} + 3 & x > 0 \\ -\frac{3}{x} - 3 & x < 0 \end{cases}$$



graph of f.

\#40 $f''(t) = 2e^t + 3\sin(t)$ $\quad f(0) = 0 \quad f(\pi) = 0$

$f'(t) = 2e^t - 3\cos(t) + C$

$f(t) = 2e^t - 3\sin(t) + Ct + D$

$f(0) = 2 + D = 0 \Longrightarrow D = -2$

$f(\pi) = 2e^{\pi} + C\pi + D = 0 \Rightarrow C\pi = 2 - 2e^{\pi}$

$C = \frac{2 - 2e^{\pi}}{\pi}$

$f(t) = 2e^t - 3\sin(t) + \frac{2(1 - e^{\pi})}{\pi}t - 2$

#44 find a function f such that

$f'(x) = x^3$ and the line $x+y=0$ is

tangent to the graph of f

$$f(x) = \frac{x^4}{4} + C \qquad y = -x \quad \text{tangent to graph.}$$

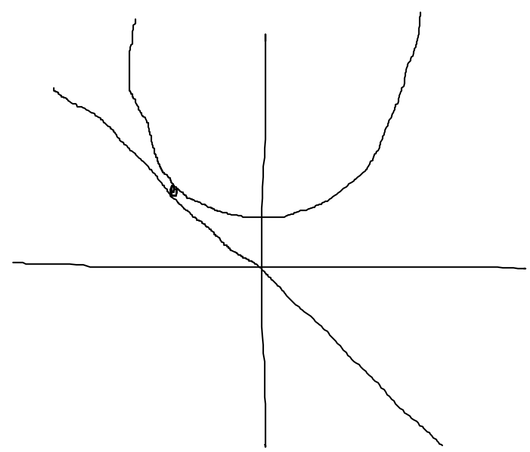
$\Rightarrow$ at some x_0,

$y = -x$

intersects the

graph at $(x_0, f(x_0))$

<u>and</u> $f'(x_0) = -1$



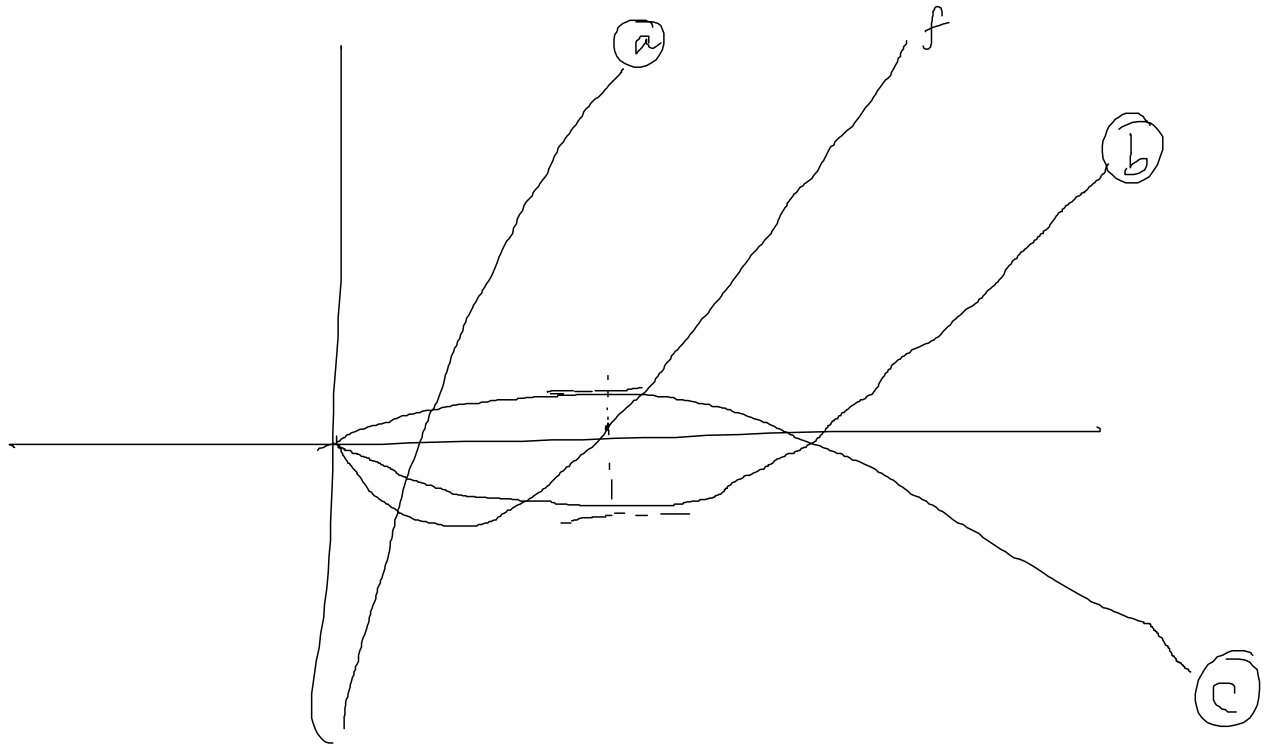
$$\Rightarrow \left(x_0, \frac{x_0^4}{4} + C\right) = (x_0, -x_0)$$

and $x_0^3 = -1 \quad \Rightarrow \quad x_0 = -1$

$$\Rightarrow \frac{x_0^4}{4} + C = -x_0 \Rightarrow \frac{1}{4} + C = 1 \Rightarrow C = 3/4$$

$$f(x) = \frac{x^4}{4} + 3/4$$

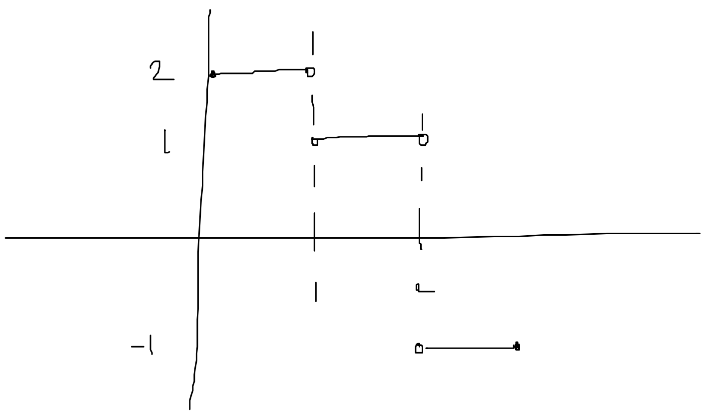
#45 the graph of f is shown.
Which of the 3 graphs is an antiderivative of f and why?

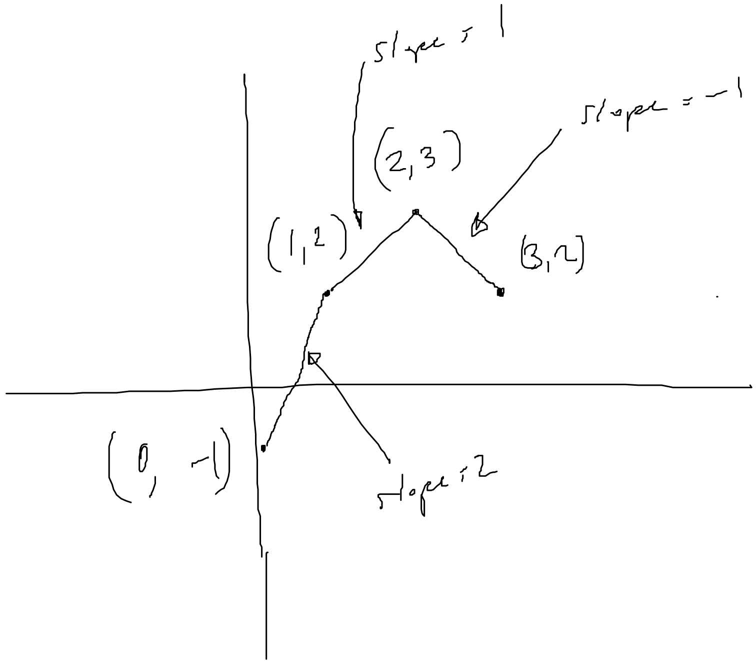


(this question would be easier if the f graph were labelled "f'" and you were asked for f.)

From the graph, f is initially negative and then becomes positive $\Rightarrow$ F will be decreasing when f is negative & will be increasing where f is positive $\Rightarrow$ it's graph (b)

#49 The graph of f' is below. Sketch the graph of f if f is continuous and $f(0) = -1$





graph of f