§ 4.5 Summary of Curve Sketching

A checklist for curve sketching

A) what is the domain of f?

B) find x- and y-intercepts

C) look for symmetries. Is f even? odd? periodic?

D) look for asymptotes (vertical, horizontal, others

E) find intervals of increase + decrease

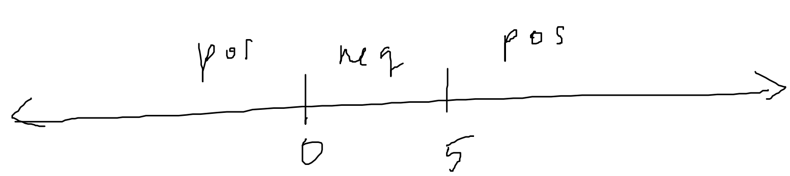
F) find local maximum & minimum values

G) find intervals of concavity & inflection points

H) finally! sketch the curve!

ex: #22 $f(x) = \sqrt{\dfrac{x}{x-5}}$

A) domain? where is the discriminant, $\frac{x}{x-5}$, ≥ 0?



$\Rightarrow$ domain $= (-\infty, 0] \cup (5, \infty)$

B) x- and y- intercepts.

x-intercepts: When does $f(x)=0$?

$$f(x)=0 \iff \sqrt{\frac{x}{x-5}} = 0 \iff \frac{x}{x-5}=0$$

$$\iff x=0$$

y-intercepts: what is $f(0)$?

$f(0)=0$.

C) symmetries: If f were even then $f(-x)=f(x)$ for all x in the domain of f. This is impossible since 1 is in the domain of f, but -1 isn't. Similarly, f isn't odd or periodic

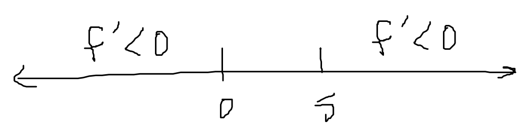
D) vertical asymptote at $x=5$
horizontal asymptote at $y=1$

E) intervals of increase & decrease:

$$f'(x) = \frac{-5}{2}\sqrt{\frac{x-5}{x}}\frac{1}{(x-5)^2}$$

we see that $f'(0)$ and $f'(5)$ DNE.

(f blows up at $x=5$ and has vertical tangent at $x=0$)



So $f' < 0$ on $(-\infty, 0) \cup (5, \infty)$

and f has critical numbers 0 & 5.

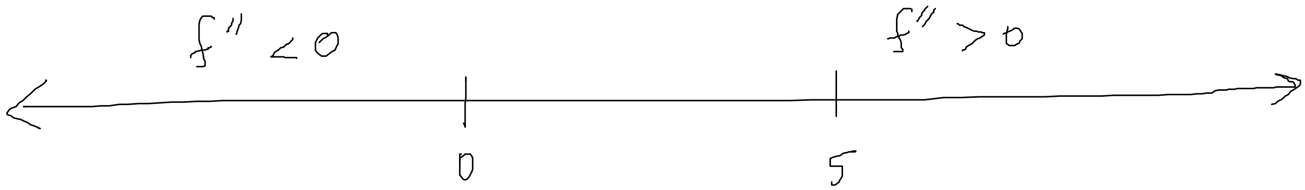
F) find local maxima & minima

There are none. (They would have to occur at critical numbers in $(-\infty, 0) \cup (5, \infty)$ and there are no such critical numbers.)

G) find intervals of concavity

$$f''(x) = \frac{-25}{4}\sqrt{\frac{x}{x-5}}\,\frac{1}{x^2(x-5)^2} + 5\sqrt{\frac{x-5}{x}}\,\frac{1}{(x-5)^3}$$

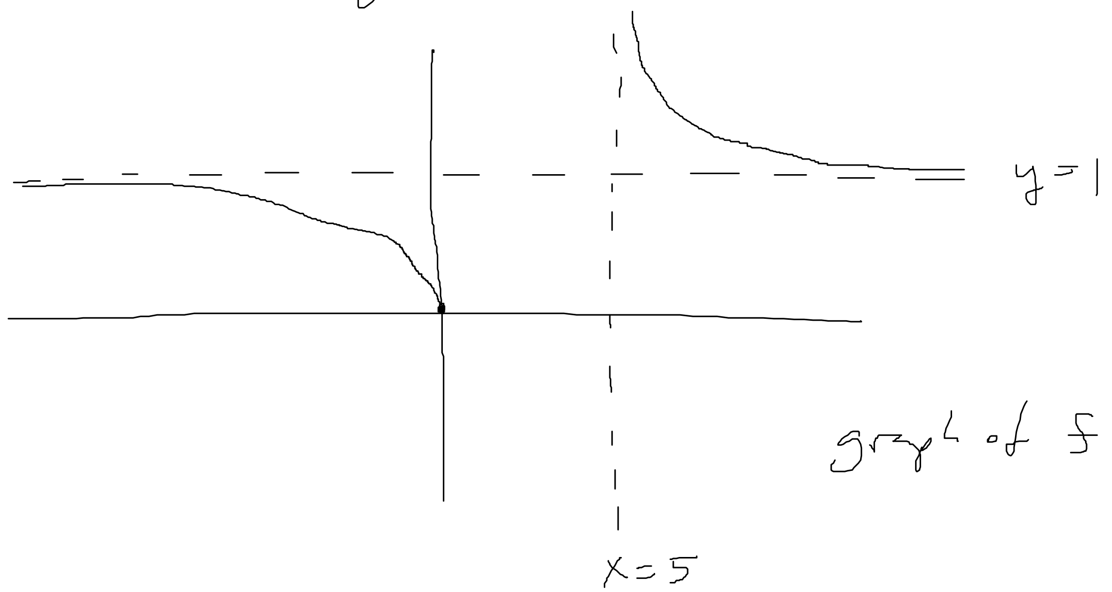
$$= \sqrt{\frac{x-5}{x}}\,\frac{1}{(x-5)^3}\,\frac{5(4x-5)}{4x}$$



graph of f concave up on $(5, \infty)$

concave down on $(-\infty, 0]$

H) We're ready to graph!



$$\lim_{x \to 5^+} f(x) = \infty$$

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ex # 39 $\quad f(x) = \dfrac{\sin(x)}{1 + \cos(x)}$

A) f is defined for all x except for when $\cos(x) = -1$. Domain $= (-\infty, \infty)$ without $\pi + n2\pi$, n any integer

B) x-intercepts. $f = 0$ whenever $\sin(x) = 0$

x intercepts at $x = 0 + n\pi$, n any integer

y-intercept $\quad f(0) = \dfrac{\sin(0)}{1 + \cos(0)} = 0$.

c) symmetries:

f(x) is 2π periodic since

$$f(x + 2\pi) = f(x)$$

f is odd since

$$f(-x) = \frac{\sin(-x)}{1 + \cos(-x)} = \frac{-\sin(x)}{1 + \cos(x)} = -f(x).$$

D) there are no y-asymptotes. there are infinitely many x-asymptotes

$$\lim_{x \to \pi-} f(x) = \lim_{x \to \pi-} \frac{\sin(x)}{1 + \cos(x)} = \infty$$

$$\lim_{x \to \pi+} f(x) = \lim_{x \to \pi+} \frac{\sin(x)}{1 + \cos(x)} = -\infty$$

E) intervals of increase & decrease

$$f'(x) = \frac{1}{1 + \cos(x)}$$

since $\cos(x) \geq -1$ for all x,

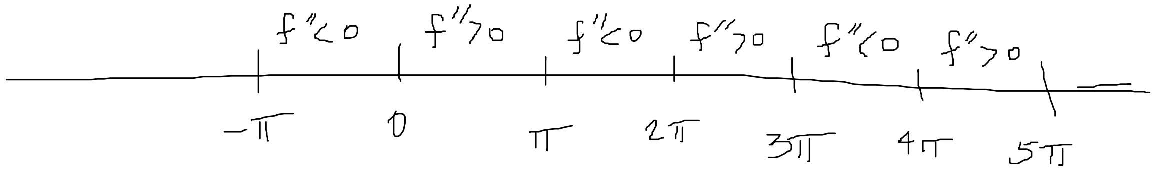
$1 + \cos(x) \geq 0$ for all x

$\Rightarrow f' > 0$ on its domain

F) local maxima & minima: there are none

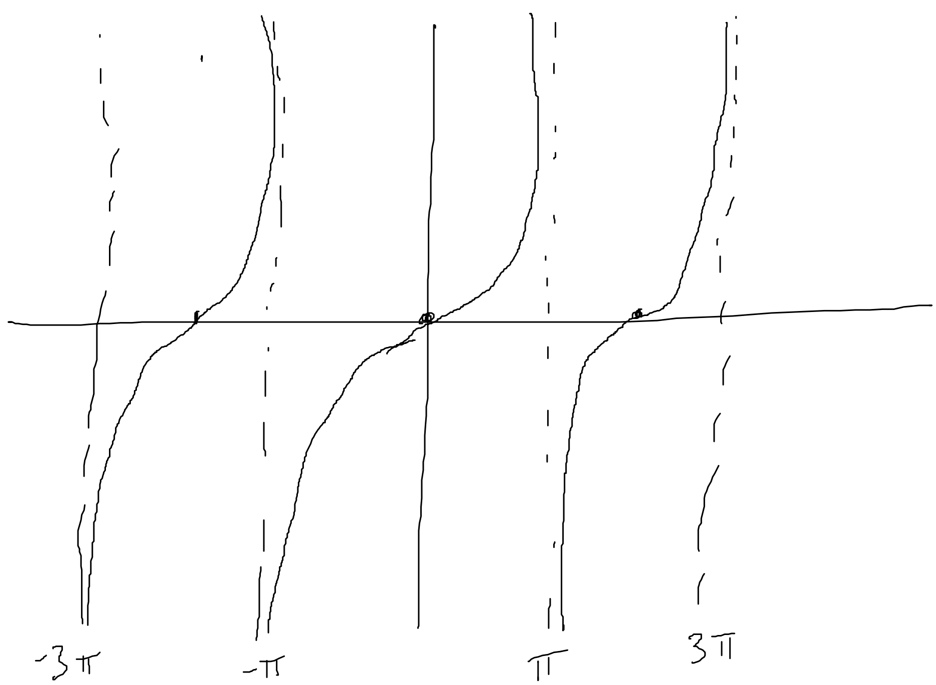
G) intervals of concavity & inflection points

$$f''(x) = \frac{\sin(x)}{(1+\cos(x))^2}$$



H) Inflection points at $0 + n \cdot 2\pi$ n any integer

Sketch the graph!



ex: $f(x) = \dfrac{2x^3 + x^2 + 1}{x^2 + 1}$

simplify with long division

$$\begin{array}{r|l} & 2x + 1 \\ x^2+1 & 2x^3 + x^2 + 0x + 1 \\ & 2x^3 \quad + 2x \\ \hline & \qquad x^2 - 2x + 1 \\ & \qquad x^2 \qquad + 1 \\ \hline & \qquad\quad -2x \end{array}$$

So $f(x) = 2x+1 - \dfrac{2x}{(x^2+1)}$

A) the domain of f is $(-\infty, \infty)$

B) x-intercepts? $f(x) = 0$ if

$$2x^3 + x^2 + 1 = 0$$

$2x^3 + x^2 + 1 = (x+1)(2x^2 - x + 1) = 0$ only at $x = -1$

y-intercepts? $f(0) = 1$

c) look for symmetries. not periodic.

$f(-x) = -2x + 1 + \dfrac{2x}{x^2+1} \neq \pm f(x)$ neither even nor odd.

D) Look for asymptotes.

No vertical asymptotes.

As $x \to \infty$ $f(x) \cong 2x+1$

As $x \to -\infty$ $f(x) \cong 2x+1$

"slant asymptote"

E) intervals of increase & decrease

$$f'(x) = \frac{2x^2(x^2+3)}{(x^2+1)^2}$$

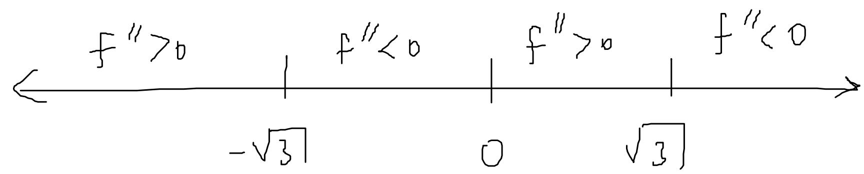
$\Rightarrow$ $f'(x) = 0$ at $x=0$ but otherwise $f'>0$.

F) local maxima & minima

none. By first derivative test, $x=0$ is neither local max nor local min

G) Intervals of concavity

$$f''(x) = -\frac{4x(x^2-3)}{(x^2+1)^3}$$



3 inflection points: $\pm\sqrt{3}$, 0

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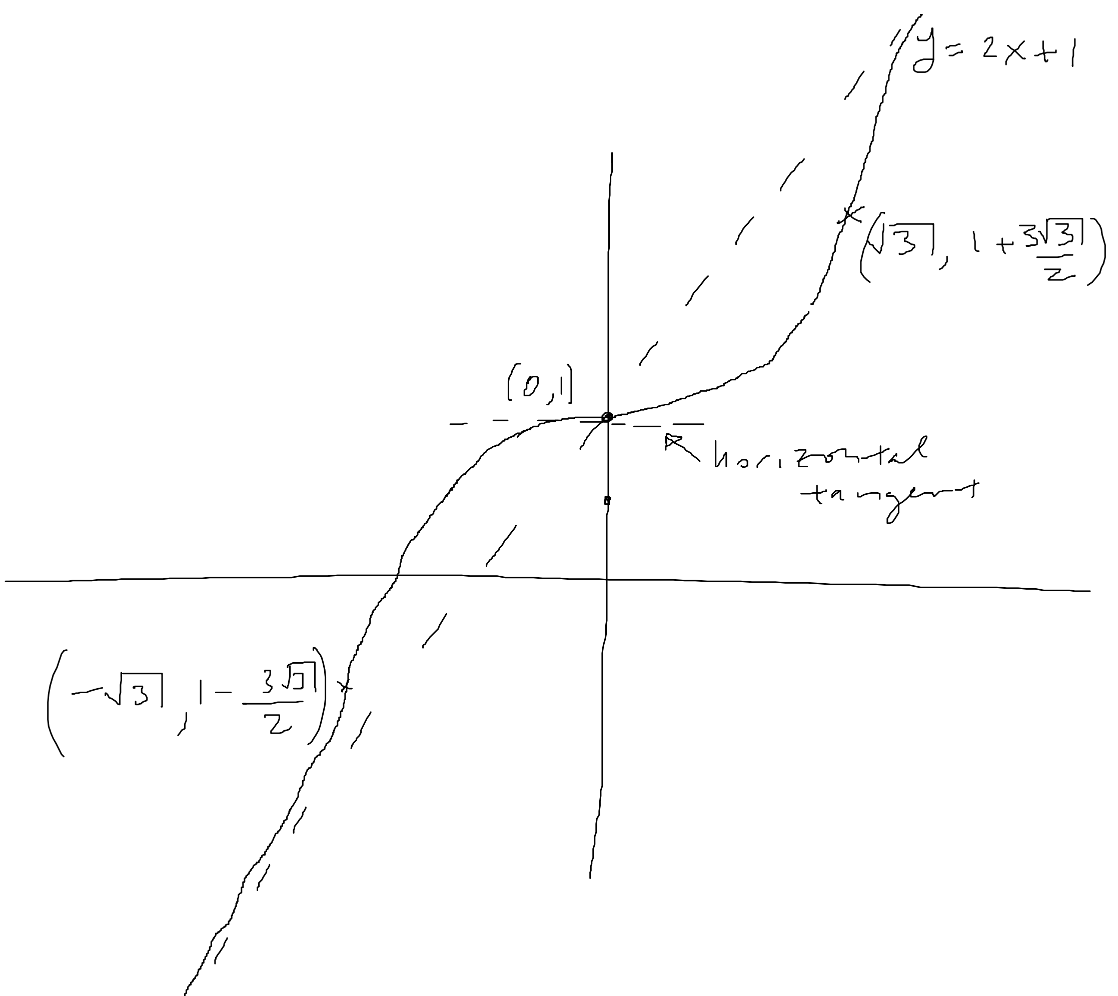
H) Sketch the graph!

$$f(-\sqrt{3}) = 1 - \frac{3\sqrt{3}}{2} \cong -1.60$$

$$f(0) = 1$$

$$f(\sqrt{3}) = 1 + \frac{3\sqrt{3}}{2} \cong 3.60$$



as $x \to \infty$ the graph of f hugs the graph of $2x+1$ tighter & tighter.

similarly as $x \to -\infty$.