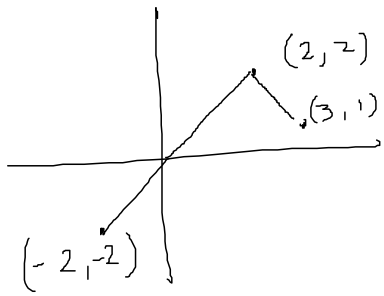
Mat 135, Nov 12 2004

From last time, a function does not have to be differentiable in order to have an absolute maximum

$-|x-2|+2$ on $[-2, 3]$



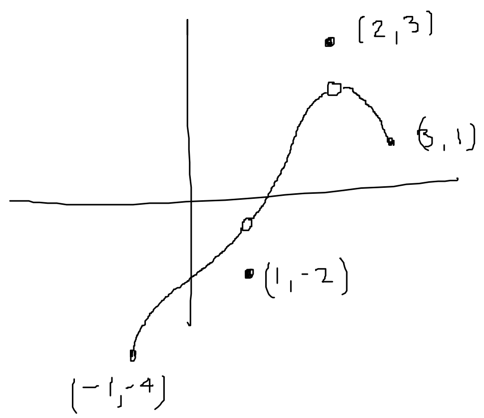
has absolute max at 2

has local max at 2

has absolute min at -2

has no local min

Also, we know the function doesn't even need to be continuous.



← graph of f on $[-1, 3]$

has absolute max at 2

has local max at 2

has absolute min at -1

has local min at 1

However, if f happens to have a local max or min at c and if $f'(c)$ exists then theorem states $f'(c)=0$.

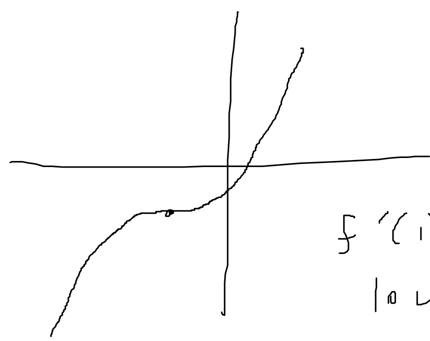
<u>Fermat's theorem</u>: If f has a local max or min at c and if $f'(c)$ exists then $f'(c)=0$.

Also, since an absolute maximum is also a local maximum, it follows that:

If f has an absolute max or min at c and $f'(c)$ exists then $f'(c) = 0$.

Note: $f'(c) = 0 \not\Rightarrow c$ is a local max or min!

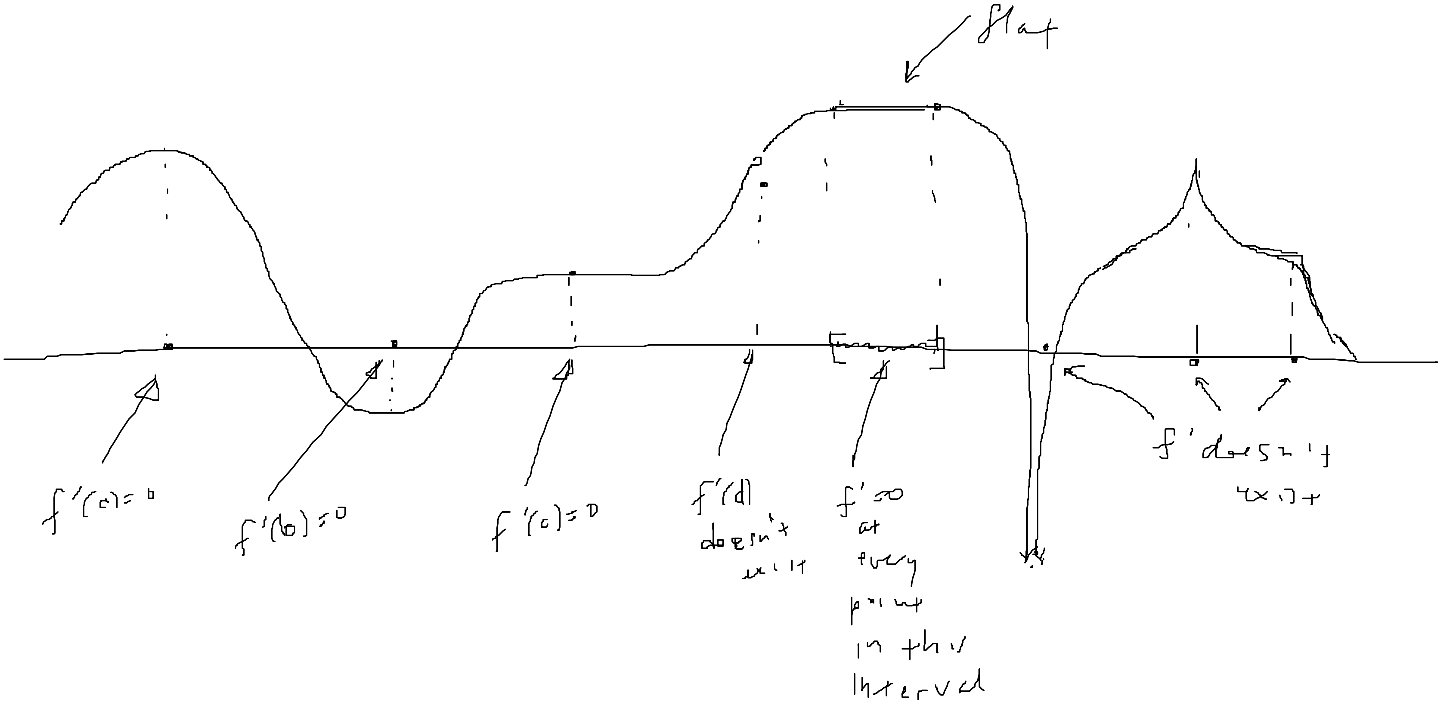
why? consider $f(x) = (x+1)^3 - 2$



$f'(1) = 0$ but 1 is neither a local max nor a local min.

definition: a critical number of f is a number c in the domain where $f'(c) = 0$ or $f'(c)$ does not exist.

critical numbers can come in all sorts of types!



<u>the extreme value theorem</u>: if f is continuous on a closed interval $[a,b]$ then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ for some numbers c and d in $[a,b]$.

<u>New fermat's theorem</u>: if f has a local maximum or minimum at c then c is a critical number of f.

Closed interval method to find local maxima and minima of a continuous function on $[a,b]$

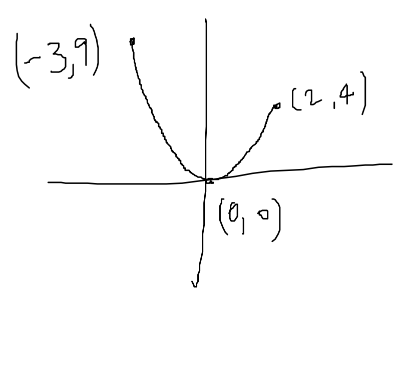
1) find all critical numbers in (a,b)
find the value of f at each of them.

2) find the values of f at the endpoints of the interval $f(a)$ & $f(b)$.

3) the largest value you find comes from the absolute maximum. The smallest value you find comes from the absolute minimum.

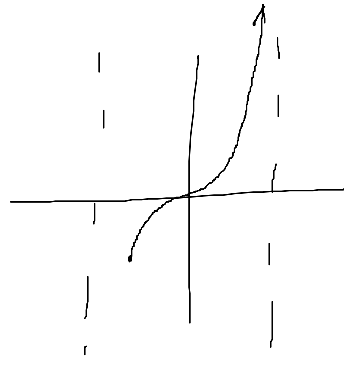
ex: 21 $\quad f(x) = x^2 \qquad -3 \le x \le 2$

critical numbers in $(-3,2)$: 0



abs max at -3
no local max
abs min at 0
local min at 0

ex: #26 $\quad \tan(x) \qquad -\frac{\pi}{4} \le x < \pi/2$



no critical numbers in $(-\frac{\pi}{4}, \pi/2)$

no abs max
no local max
abs min at $-\pi/4$
no local min

49 $f(x) = 2x^3 - 3x^2 - 12x + 1$ on $[-2, 3]$

f' exists everywhere. to find critical numbers in $(-2,3)$, we just need to find where in $(-2,3)$ $f'(c) = 0$.

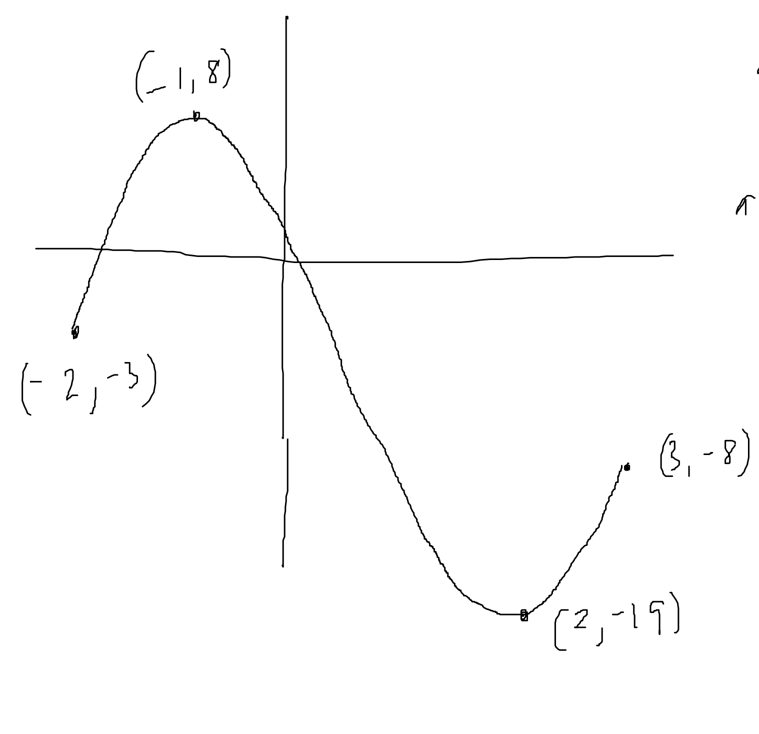
$$f'(x) = 6x^2 - 6x - 12$$
$$= 6(x^2 - x - 2)$$
$$= 6(x-2)(x+1)$$

critical numbers in $(-2,3)$: -1 and 2

$$f(-1) = 8$$
$$f(2) = -19$$

endpoints

$$f(-2) = -3$$
$$f(3) = -8$$



abs max at -1

local max at -1

abs min at 2

local min at 2

#54 $f(x) = \dfrac{x^2 - 4}{x^2 + 4}$ on $[-4, 4]$

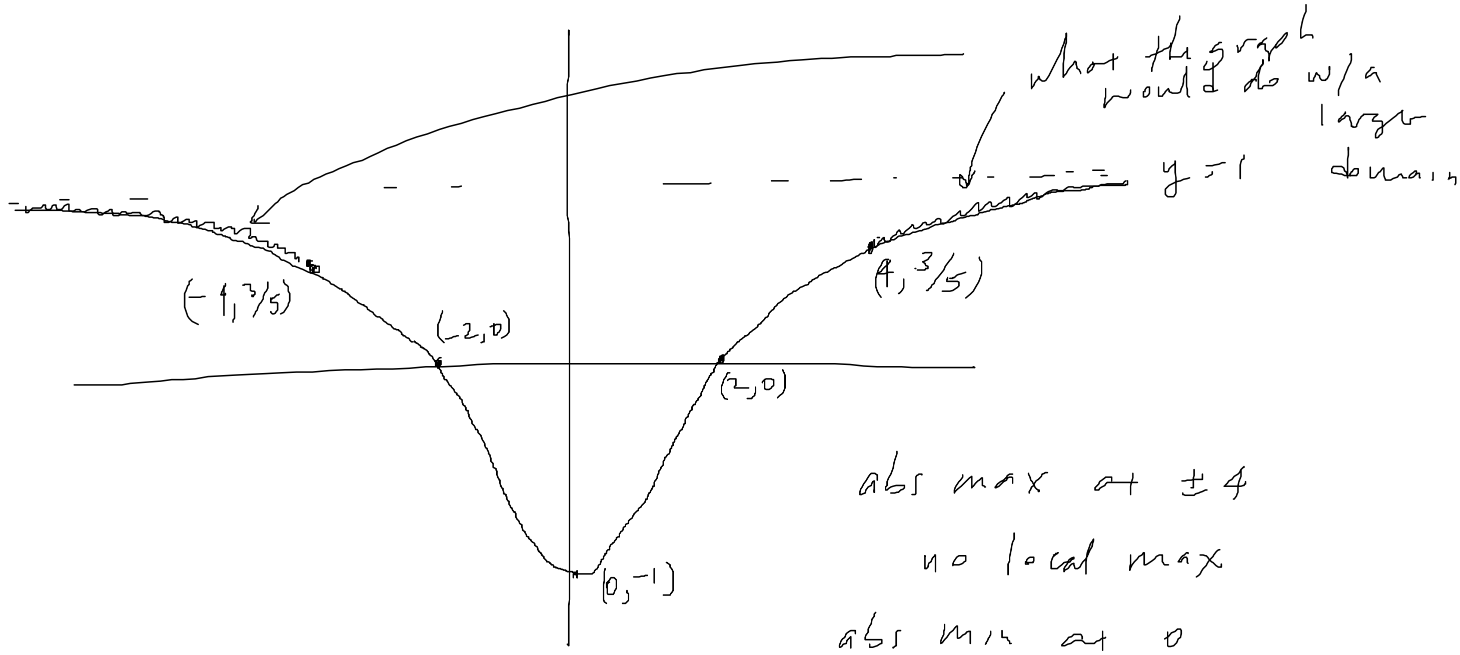
$$f'(x) = \frac{16x}{(x^2+4)^2}$$

there's only one critical number in $(-4, 4)$: 0

$f(0) = -1$

endpoints $\to$ $f(-4) = +3/5$

$f(4) = +3/5$



abs max at ± 4

no local max

abs min at 0

local min at 0

#61 $f(x) = x - 3\ln x$ on $[1, 4]$

$f' = 1 - \frac{3}{x}$

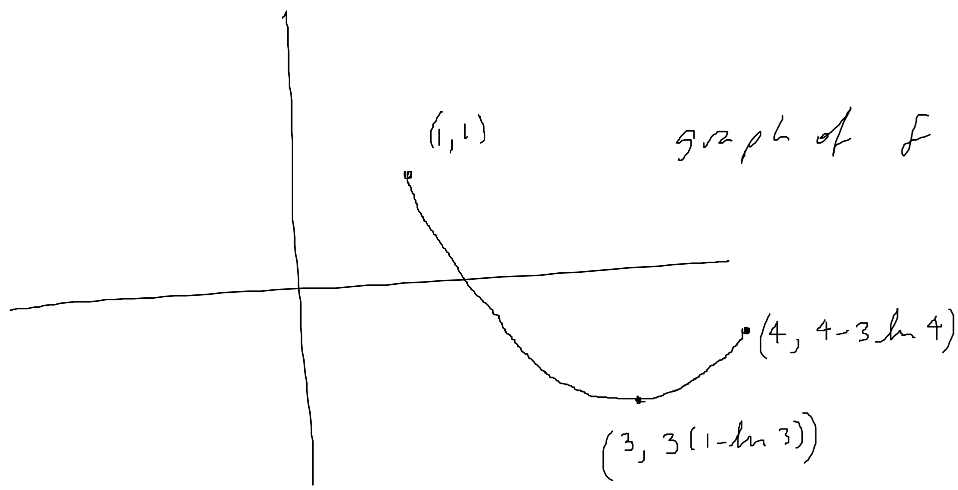
one critical number in $(1, 4)$: 3

$f(1) = 1$

$f(3) = 3(1 - \ln(3)) \cong -0.296$

endpoints

$f(4) = 4 - 3\ln 4 \cong -0.159$



abs max at 1

no local max

abs min at 3

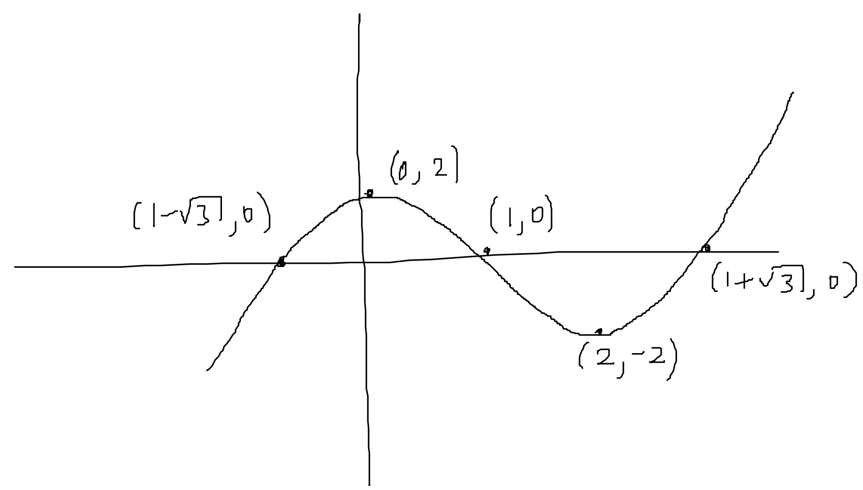
local min at 3

64 find the critical numbers of $f(x) = |x^3 - 3x^2 + 2|$

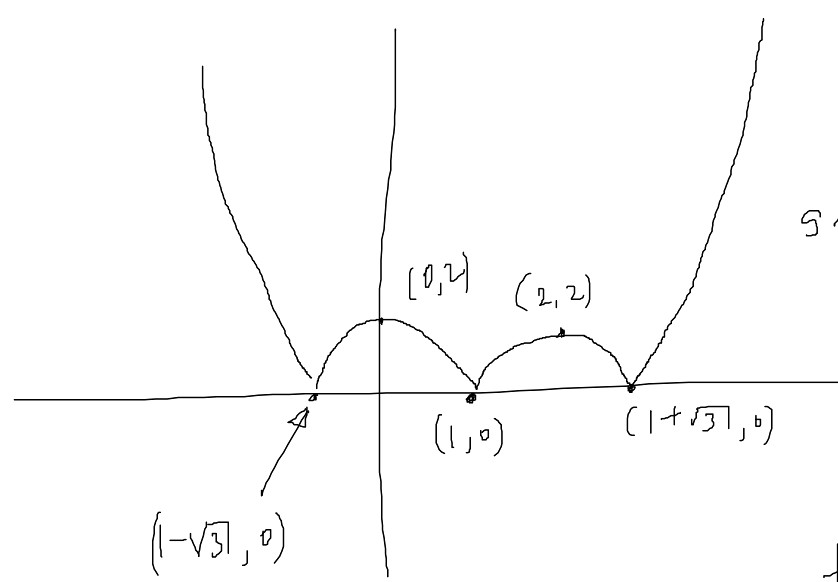
first, plot $g(x) = x^3 - 3x^2 + 2$

$$= (x-1)(x-(1+\sqrt{3}))(x-(1-\sqrt{3}))$$

$$g'(x) = 3x^2 - 6x = 3x(x-2)$$



graph of g



graph of f

critical numbers:

$f'(c) = 0$ at $0, 2$

$f'(c)$ does not exist.
at $1, 1+\sqrt{3}, 1-\sqrt{3}$