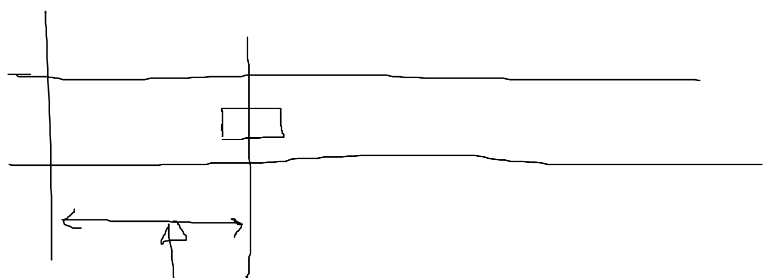
Mat 135 Dec 1, 2004

Antiderivatives continued.

II

position, velocity, acceleration



0 $s(t)$ = distance from the starting line at time t.

$\Rightarrow \frac{ds}{dt}$ = velocity of car

$\Rightarrow \frac{d^2s}{dt^2}$ = acceleration of car

#60 a particle is moving w/ the given data. Find its position

$v(t) = \frac{3}{2}\sqrt{t}$. $s(4) = 10$

$\frac{ds}{dt} = \frac{3}{2}\sqrt{t} \Rightarrow s(t) = t^{3/2} + C$

$s(4) = 10 \Rightarrow 4^{3/2} + C = 10 \Rightarrow C = 2$

and so $\boxed{s(t) = t^{3/2} + 2}$

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$$a(t) = \cos(t) + \sin(t) \qquad s(0) = 0 \quad v(0) = 5$$

$$a(t) = \frac{dv}{dt} = \cos(t) + \sin(t)$$

$$\Rightarrow v(t) = \sin(t) - \cos(t) + C$$

$$v(0) = \sin(0) - \cos(0) + C = 5 \Rightarrow C = 6$$

$$v(t) = \sin(t) - \cos(t) + 6$$

$$v(t) = \frac{ds}{dt} \qquad \Rightarrow s(t) = -\cos(t) - \sin(t) + 6t + D$$

$$s(0) = -\cos(0) - \sin(0) + 6 \cdot 0 + D = 0$$

$$\Rightarrow D = 1$$

$$\boxed{s(t) = -\cos(t) - \sin(t) + 6t + 1}$$

#74 A car is travelling at 50 mi/h when the brakes are fully applied, causing a constant deceleration of 22 ft/s^2

What is the distance travelled before the car comes to a complete stop?

given: $a(t) = -22 \ \frac{ft}{sec^2}$

$$\Rightarrow a(t) = \frac{dv}{dt} = -22 \ ft/sec^2$$

$$\Rightarrow v(t) = -22t + C \quad \frac{ft}{sec}$$

$v(0) = 50 \frac{mi}{h}$ ← need to convert to feet/sec in order to apply initial data

$$\frac{50 \text{ mi}}{hr} \times \frac{5280 \text{ feet}}{mi} \times \frac{hr}{60 \text{ min}} \times \frac{min}{60 \text{ sec}} = 73\tfrac{1}{3} \ \frac{feet}{sec}$$

$$v(0) = -22 \cdot 0 + C = 73\tfrac{1}{3} \Rightarrow C = 73\tfrac{1}{3}$$

$$\Rightarrow v(t) = -22t + 73\tfrac{1}{3} \quad \text{ft/sec}$$

When is $v(t) = 0$? $\Rightarrow -22t + 73\frac{1}{3} = 0$

$$\Rightarrow t = \frac{73\frac{1}{3}}{22} \text{ sec}$$

$$= \frac{10}{3} \text{ sec}$$

How far has the car travelled?

$$v(t) = \frac{ds}{dt} \Rightarrow s(t) = -11t^2 + 73\tfrac{1}{3}t + D \text{ feet.}$$

$$s(0) = D \text{ feet}$$

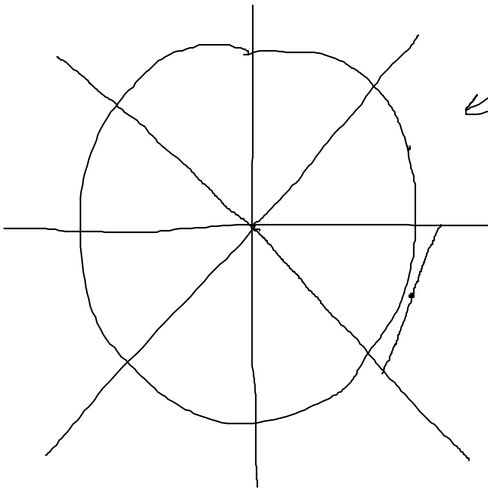
$$s\left(\frac{10}{3}\right) = -11\left(\frac{10}{3}\right)^2 + \left(73\tfrac{1}{3}\right)\left(\frac{10}{3}\right) + D = \frac{1100}{9} + D \text{ feet}$$

distance travelled while stopping

$$= s\left(\frac{10}{3}\right) - s(0) = \frac{1100}{9} \text{ feet} \cong 122 \text{ feet}$$

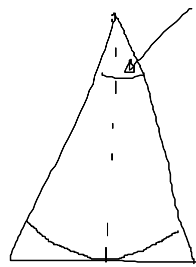
§5.1 Areas & Distances.

You know the area of a rectangle & a triangle.
How do you find the area of other objects?



circle of radius r

Approximate with 8 triangles

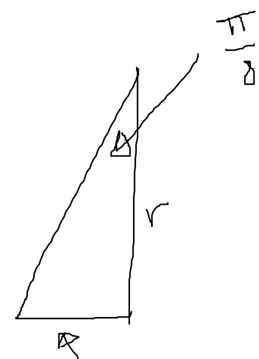


$\theta = \frac{2\pi}{8}$

height $= r$

know the triangle.

$$\text{area} = 8\left(\frac{1}{2}\,\text{height}\cdot\text{base}\right)$$



$$\tan\left(\frac{\pi}{8}\right) = \frac{\text{opp}}{r}$$

$$\Rightarrow \text{opp} = r\tan\left(\frac{\pi}{8}\right)$$

$\Rightarrow$ base

$$= 2\,\text{opp}$$

$$= 2r\tan\left(\frac{\pi}{8}\right)$$

triangle area

$$= 8\left(\frac{1}{2}r\right)\left(2r\tan\left(\frac{\pi}{8}\right)\right)$$

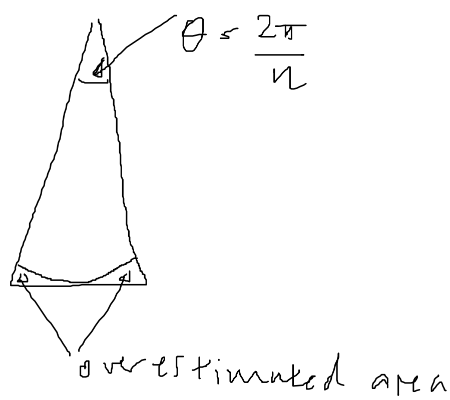
$$= 8r^2\tan\left(\frac{\pi}{8}\right)$$

If we'd approximated the circle with

n triangles then

$$\text{approximate area of circle} = n r^2 \tan\left(\frac{\pi}{n}\right)$$

note: the approximate area will always be a little too big since we're always overestimating



but the larger n is, the smaller the overestimate is

$$\lim_{n\to\infty} r^2 n \tan\left(\frac{\pi}{n}\right)$$

let $x = \frac{1}{n}$

then $n \to \infty$

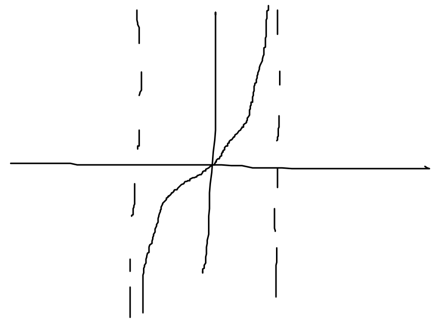
corresponds to

$x \to 0+$

$$= \lim_{x\to 0+} r^2 \frac{\tan(\pi x)}{x}$$

numer $\to 0$

den $\to 0$



$\Rightarrow$ apply

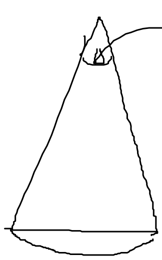
l'Hospital's rule

$$= \lim_{x\to 0+} r^2 \frac{\sec^2(\pi x)\cdot\pi}{1}$$

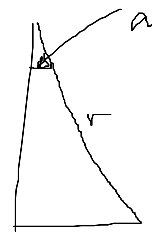
$$= \lim_{x\to 0+} \pi r^2 \frac{1}{\cos^2(\pi x)} = \pi r^2$$

We just proved that the area of a circle is πr^2!

we could've also done this by underestimating the area:



angle $= \frac{2\pi}{n}$

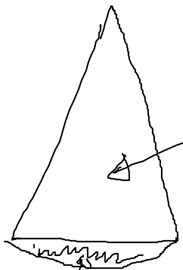


angle $= \pi/n$

r

height $= r\cos\left(\frac{\pi}{n}\right)$

base $= r\sin\left(\frac{\pi}{n}\right)$



$$\text{area} = \frac{1}{2}\left(2r\sin\left(\frac{\pi}{n}\right)\right)\left(r\cos\left(\frac{\pi}{n}\right)\right)$$

your estimate is missing this.

$$= r^2 \sin\left(\frac{\pi}{n}\right)\cos\left(\frac{\pi}{n}\right)$$

area of circle underestimated by

$$n\left[r^2\sin\left(\frac{\pi}{n}\right)\cos\left(\frac{\pi}{n}\right)\right]$$

$$\lim_{n\to\infty} nr^2\sin\left(\frac{\pi}{n}\right)\cos\left(\frac{\pi}{n}\right)$$

$x = \frac{\pi}{n}$ $\quad n \to \infty$ then $x \to 0^+$

$$= \lim_{x\to 0^+}\left(\frac{\pi}{x}\right) r^2 \sin(x)\cos(x)$$

$$= \pi r^2 \lim_{x\to 0^+} \frac{\sin(x)}{x}\cos(x)$$

$$= \pi r^2$$