Homework due Monday December 6, in my mailbox by 5pm.

1. **Integral Solutions** Consider the Riemann problem given by (53) on page 154. Formula (56) is alleged to be an integral solution. Prove this is true.

2. **the Lax-Oleinik formula** Formulas (54) and (56) are explicit representations of entropy integral solutions for the Riemann problem (53). On the other hand, the Lax-Oleinik formula (29) gives an explicit representation of the entropy integral solution. By Theorem 3 on page 151, these two representations must agree up to a set of measure zero. Prove this by explicitly computing (29) for Riemann initial data in the special case of Burger's equation: $F(z) = x^2/2$.

3. **That factor of 6 in the KdV equation** In class, I claimed that if you have a solution $u(x,t)$ of
$$u_t + 6uu_x + u_{xxx} = 0$$
then you can transform it into a solution $v(x,t)$ of
$$v_t + Avv_x + v_{xxx} = 0.$$
Find this transformation. Now that you've found it, find the soliton solutions for the v-equation.

4. **Figure on page 178.** On page 178 of the book, Evans sketches the level sets for the Lyapunov function $E(v,w)$. Fully explain what the level sets look like in the region $\{0 \leq v \leq 1, w \geq 0\}$.

5. **Stable manifolds.** On page 178 of the book, line -7, Evans writes, "Similarly, we argue W^s must hit L at a point $(a+\epsilon, w_q(\sigma))$." Present this argument.

6. **Analogous reaction-diffusion equations.** What can you say about travelling wave solutions for the reaction diffision equation $u_t = u_{xx} + f(u)$ if the reaction term satsifies conditions (13a), (13b), and (13c) on page 176 but
$$\int_0^1 f(z)\,dz < 0?$$

7. **Another travelling wave equation**. Consider the PDE
$$u_t = u_{xx} + u(1-u)$$
on $\mathbb{R} \times (0,\infty)$. Show that you can find travelling wave solutions with any speed $\sigma \geq 2$.