# Department of Mathematics, University of Toronto MAT267H1S - Advanced Ordinary Differential Equations

Syllabus—Winter 2019 January 4, 2019 revised March 15, 2019

This is an introductory course in differential equations designed for second-year MAT157-stream Specialists in the Faculty of Arts & Science (Math & Physics, Math & Philosophy, Applied Math, and Math).

The prerequisites are MAT240H1F and MAT247H1S. MAT257Y1 is a co-requisite. You need to fill out a prerequisite waiver if you do not already have course credit for both MAT240H1F and MAT247H1S or if you're not currently enrolled in MAT257Y1 (and don't already have course credit for it). Go to http://www.math.toronto.edu/cms/undergraduate-program/current-students-ug/forms/ for more information. You're welcome to attend lectures and tutorials until your prerequisite application has been decided upon.

As of mid-December approximately a third of the enrolled students were not second-year students. And approximately a third of were not in a MAT157-stream Specialist program in the Faculty of Arts & Science. All students are welcome (assuming they have the prerequisites or a waiver) and having a non-homogeneous class should make for a better learning experience. I am confident that all of you will be kind to and generous with one another.

## Lecture Information

Time & Room	Professor	Office	email address		
Tue 1:10-2, Thu 1:10-3, WB 116	Mary Pugh	BA 6268	mpugh@math.utoronto.ca		

# Webpage, Email Information

The course website is http://www.math.utoronto.ca/mpugh/Teaching/MAT267\_19/mat267. html. All announcements and handouts will be posted there. Please go there regularly. From time to time, I may contact you by via email. It is crucial that your email on ROSI be a utoronto.ca email address — otherwise you may miss course emails. If you want to email me, you must do so from your utoronto.ca email address; this is a UofT policy.

## Textbook and Additional Material

The main textbook is the third edition of *Differential Equations, Dynamical Systems, and an Introduction to Chaos* by Morris Hirsch, Stephen Smale, and Robert Devaney. A supplementary source is *Ordinary Differential Equations* by Morris Tenenbaum and Harry Pollard. Both books should be available in the UofT bookstore.

The Tenenbaum & Pollard book is a great source of worked examples, insight, and exercises. That said, it's missing some of the material that the course needs. The book is from 1963 — before the revolution in nonlinear dynamics and dynamical systems. A revolution that Smale and Hirsch were key players in. Devaney is a world-class player in the second-wave. That said, Tenenbaum & Pollard is pre-Wolfram Alpha and it contains the no-longer-glamourous-but-still-vital stuff that is

needed to really master the course material. (You cannot really master the material without rolling up your sleeves and getting your hands dirty.)

The Hirsch, Smale, & Devaney book is a more introductory version of the Hirsch & Smale classic *Differential Equations, Dynamical Systems, and Linear Algebra*. It provides the viewpoint needed to enter into the arena of nonlinear dynamics and dynamical systems. That said, it does not contain some of the analytical material that you will need in subsequent courses in geometry, analysis, and PDE. For this reason, additional material will be provided when needed.

#### Office Hours

I will hold weekly office hours. The days and times will be posted in the next week or two.

## **Tutorials**

This is a fast-paced course, and regular attendance in your tutorial is an **essential** component of this course. Every student needs to register in a tutorial section. Tutorials begin on January 18 (the 2nd week of classes). During tutorials your TA will answer your questions about the course material and discuss some problems from the suggested homework problems (see the Schedule and Suggested Problems posted on the website). Consequently, to get the most out of your tutorial you must keep up to date with the homework and come prepared to ask questions, should you have any.

Section	Time & Room					
TUT0101	Fri 1:10-2, RW 140					
TUT0201	Fri 3:10-4, SS 2127					

#### Quizzes and Homework

There will be five quizzes and six homework assignments. The first quiz will be during your first tutorial on January 18. You are to take the quiz in the tutorial you are registered for. The quiz problems are intended to test your basic understanding of the mechanics of the material and to keep you on schedule. Quizzes will be marked on a scale of 0, 1, 2, and 3.

If you are a student who feels that the mechanics of the material are not to be taken seriously (e.g. that it's sufficient to know what an eigenvalue is, rather than finding, using, and studying it) then you will dislike many of the assessments in this course. My experience is that in every research mathematician's office has a) a blue bin full of scratch work, b) notebooks full of scratch work, or c) computer directories full of photos of blackboard scratch work. I don't know of any mathematics that doesn't require scratch work, computations, and worked examples. It's not all abstract thought and "Aha!" moments. In this direction, there will be material that I expect you to teach yourselves; material that is only lightly touched upon in class. There are certain things that are best learnt by doing. And part of the quizzes and exams will test your foundational/mechanical/computational skills.

If you're having problems with the quizzes then this should be a red flag for you: either you're not engaging with the course and the material sufficiently (come see me if you don't understand what this means or if you need help) or you're not ready for this course and you should drop it. The drop date, March 17, is after both midterm exams. If you don't need MAT267H1 for your

programme, bear in mind that there are three sections of MAT244H1 being offered this semester. The deadline to add yourself to that course is January 20.

There will be no make-up quizzes. Your lowest quiz mark will be dropped and the remaining ten marks will be used towards your course mark.

# Midterm Exams

There will be two fifty-minute in-class exams. If you're enrolled in another class that also meets on Tuesdays 1:10-2 then you're out of luck if that class happens to have an in-class exam on the same day as ours. Information about what the exams will cover will be posted at least one week before the exam dates. There will be no make-up exams.

Each exam will have three problems. Two of them will be related to homework problems and should be straightforward if you did and understood the homework. The third problem may or may not be related to homework problems; it'll be more challenging.

	Date & Time	Location
Exam 1	Tuesday February 5, 1:10-2pm	WB 116
Exam 2	Tuesday March 12, 1:10-2pm	WB 116

# Communication and In-class Participation

Five percent of your course mark will be determined by an essay. This essay should be 2-5 pages long. It should be about some ordinary differential equation (or system of ODEs) that you are curious to learn more about. Either because it's inherently interesting to you or because it will be important in some area of science or mathematics that you care about. You'd present the ODE(s) and explain why it's interesting to you or what it's relevant to and give a summary of what you've learnt about it to date. If you don't have a clue what to write about, you could try googling. You could join the math union facebook page https://www.facebook.com/groups/Math.Union/ and ask other students how ODEs have popped up in their courses. You could ask random undergrads, graduate students, postdocs, and faculty members on the 6th floor of Bahen.

You can submit the essay at any time on or before the last day of class (April 4). In fact, you can submit it three times — I will happily read your first version and give feedback and a grade. If you'd like to improve your first version, I will happily read your second version and give feedback and a grade. You can then use that, should you wish, to improve your second version and submit your final version. I am not looking for poetry or great literature. I am looking for your own thoughts and work (see below for academic integrity reminders). If you are nervous about writing, feel free to come to my office hours and show me a sample and I'll let you know if I have problems understanding your writing. And, of course, all of the colleges have Writing Centres http://writing.utoronto.ca/writing-centres/arts-and-science/ where you can get writing support. (If you're an FAS student you're a member of a College; that's your Writing Centre. If you're not an FAS student then you should have access to writing supports at your home campus or Faculty.)

Being able to speak and write easily and clearly is a vital life skill. Thriving in math classes relies on your creative problem-solving skills and your analytical abilities. These are eternally valuable (and somewhat rare) but are of little use if you can't explain your thinking, your process, and your solution! On occasion, I will ask for student volunteers to come to the blackboard and present their ideas or work. If you're avoiding volunteering because you're nervous speaking in public, you've got company. If you're nervous because you don't feel confident enough about your English, consider checking out the Communication Cafe http://www.artsci.utoronto.ca/current/advising/ell/communication-cafe. If you're happy with your grammar but worry about being articulate or how to best present your ideas, consider joining a local Toastmaster's group. https://www.toastmasters60.com/find-a-club/. This is a legitimate, nearly-century-old organization¹ https://en.wikipedia.org/wiki/Toastmasters\_International.

## Crowdmark

We will be using crowdmark to mark the midterm exams, and the final exam. This means that your exams will scanned and the resulting pdf file will be uploaded onto the crowdmark servers. Crowdmark is not out-sourcing grading — your quizzes and exams will be graded by the TAs and me. If you wish to opt out of having your work marked via crowdmark, please let us know and we will manually grade your work.

# Remarking Procedure

Should you feel that a quiz or exam has been misgraded, then you must submit a request for regrading (with written justification) within 7 days of when the materials were returned<sup>2</sup>. Your TAs will handle quiz regrades and I will handle exam regrades. If you find that you are not satisfied by the outcome of your remarking request for a quiz then you can make a follow-up request to have me remark the problem in question. In all cases, bear in mind that a remarking request can lead to a higher mark, a lower mark, or no change in mark. Some instructors would say that quizzes cannot be submitted for remarking if they were written in pencil rather than pen. I like pencils and so I will simply tell you that I know an undergrad who modified his quizzes, submitted them for remarking, and was suspended for six months as a result. For quizzes that were worth less than 2% each. I'm confident that you are honourable students and so this bit of information is irrelevant.

#### Missed Term Work

If you miss a quiz, don't worry about it — your lowest quiz mark is being dropped.

If you miss an exam for a legitimate and serious reason, you or someone who speaks for you must email me within twenty-four hours of the exam. In addition, you must submit a hard copy the standard documentation to me within one week of the exam. Go to http://www.artsci.utoronto.ca/current/petitions/process#documentation for instructions and more information.

<sup>&</sup>lt;sup>1</sup>When Rotman, CAMH, Engineering, Sick Kids, and Queens Park all have toastmaster clubs, you should know there's a there there. Unless you're intrigued by other groups, or want one close to where you live, consider one of the two engineering ones? Or just choose one that's open and has a meeting time that works for you.

<sup>&</sup>lt;sup>2</sup>Note that that if you don't pick your quizzes up at tutorial then you may miss out on the remarking window — if you pick all of them up at the end of the semester you'll have likely missed the remarking window for all but the last quiz.

## Life can be complicated

Everyone is different. Each of us has our own strengths, weaknesses, gifts, and needs. Life would be monochrome were it otherwise. If there are accommodations that would help you achieve academic success in this course, please feel free to approach me or Accessibility Services https://www.studentlife.utoronto.ca/as, if you haven't already done so.

From https://www.studentlife.utoronto.ca/asc/hours: "Would you like to

- Learn to manage time and address procrastination issues and stress?
- Develop new strategies, including active studying, reading and note-taking, and exam preparation?
- Improve your research, writing, and presentation skills?
- Learn about the University's academic systems and services?"

A learning strategist can help you with this! You can make an appointment to see one at the Student Success Centre; they're also at a variety of locations around campus including First Nations House and all seven colleges. Go to the above URL for more information.

If you are experiencing challenges that are having an impact on your academic work, please seek support sooner rather than later. Your college registrar should be able to help. For some quick resources re: academic/financial/housing/mental health distress or sexual assault/safety, see https://studentlife.utoronto.ca/feeling-distressed.

#### Collaboration

Working together on course material, including homework, is a useful and mathematically healthy practice. I strongly encourage you to find some study buddies! However, you need to make sure that you fully understand the material yourself. If you're with others and you solve a HW problem as a team then step back and return to the problem in a few hours. Without looking at your notes solve the problem again and write up the solution. If you can't do this you've discovered two things: 1) you haven't understood the material sufficiently well to be able to generate the solution on your own and 2) if you were to look at the notes and use them to write up a solution and submit that for a mark, you would be submitting someone else's work in your name. This is an academic offense! (We're not collecting and marking HW in this course so it's a moot point for us, but the point stands.)

As you become more advanced in mathematics, or any academic research, you will be expected to be able to identify who provided what ideas/techniques and you'll be allowed to provide work with multiple authors. But for this course the standard is: you should be using your own words and it should be your own work in that you were able to write it up without looking at/speaking to other sources. (It's fine if you didn't invent the wheel as long as you know how to build one, how to use one, and why it works.)

From http://www.artsci.utoronto.ca/newstudents/transition/academic/plagiarism, "Honesty and fairness are considered fundamental values shared by students, staff and faculty at the University of Toronto. The University's policies and procedures that deal with cases of cheating, plagiarism (representing someone else's work as your own), and other forms of academic misconduct are designed to maintain a community where competition is fair. The vast majority of students are honest and hard-working. But sometimes even honest people make bad decisions and accidents

sometimes happen. Even if you think you know the rules, double-check. The consequences of not knowing the rules can be severe, and include failed courses, suspension, and in very serious cases permanent expulsion." For more information, please read the University policy on academic misconduct at the above webpage.

I take academic integrity very seriously. If you have any questions at any time, about this course or any other, please ask! For this course, I'm confident you know how to behave during quizzes and exams. If you're worried about the essay, just come talk to me (or talk to someone at a Writing Centre). Reading http://advice.writing.utoronto.ca/using-sources/how-not-to-plagiarize/ may be a good start.

# Marking Scheme

On February 28, the class discussed the assessment scheme for the course and various alternate blends were proposed in the place of the original blend course work. After a fairly lengthy and thorough discussion, twelve blends were voted on — a student's marks will be combined according to each blend and then the maximum will be taken:

Course Mark =  $\max\{\text{blend } 1, \text{ blend } 2, \dots \text{blend } 12\}$ 

Blend 1	Blend 2	Blend 3	Blend 4	Blend 5	Blend 6	Blend 7*	Blend 8	Blend 9	Blend 10	Blend 11	Blend 12
			6%	6%	6%	6%	5%	9%	9%		9%
6%	6%	6%				6%	6%	6%	6%		6%
30%	30%	24%	27%	30%	25%	25%	27%	25%	25%	30%	35%
19%	15%	15%	22%	19%	20%	15%	17%	15%	15%	15%	
40%	44%	50%	40%	40%	44%	43%	35%	35%	40%	45%	45%
5%	5%	5%	5%	5%	5%	5%	10%	10%	5%	10%	5%
						* all 6 HW					
	6% 30% 19% 40%	6% 6% 30% 30% 19% 15% 40% 44%	6% 6% 6% 30% 30% 24% 19% 15% 15% 40% 44% 50%	6% 6% 6% 6% 30% 30% 24% 27% 19% 15% 15% 22% 40% 44% 50% 40%	6% 6% 6%   6% 6% 6%   30% 30% 24% 27% 30%   19% 15% 15% 22% 19%   40% 44% 50% 40% 40%	6% 6% 6% 6%   6% 6% 6% 6%   30% 30% 24% 27% 30% 25%   19% 15% 15% 22% 19% 20%   40% 44% 50% 40% 40% 44%	6% 6% 6% 6% 6%   6% 6% 6% 6% 6% 6%   30% 30% 24% 27% 30% 25% 25%   19% 15% 15% 22% 19% 20% 15%   40% 44% 50% 40% 40% 44% 43%   5% 5% 5% 5% 5% 5%	6% 6% 6% 6% 5%   6% 6% 6% 6% 6% 6%   30% 30% 24% 27% 30% 25% 25% 27%   19% 15% 15% 22% 19% 20% 15% 17%   40% 44% 50% 40% 40% 44% 43% 35%   5% 5% 5% 5% 5% 5% 5% 10%	6% 6% 6% 6% 5% 9%   6% 6% 6% 6% 6% 6%   30% 30% 24% 27% 30% 25% 25% 27% 25%   19% 15% 15% 22% 19% 20% 15% 17% 15%   40% 44% 50% 40% 40% 44% 43% 35% 35%   5% 5% 5% 5% 5% 5% 10% 10%	6% 6% 6% 6% 5% 9% 9%   6% 6% 6% 6% 6% 6% 6%   30% 30% 24% 27% 30% 25% 25% 27% 25% 25%   19% 15% 15% 22% 19% 20% 15% 17% 15% 15%   40% 44% 50% 40% 40% 44% 43% 35% 35% 40%   5% 5% 5% 5% 5% 10% 10% 5%	6% 6% 6% 6% 5% 9% 9%   6% 6% 6% 6% 6% 6% 6%   30% 30% 24% 27% 30% 25% 25% 27% 25% 25% 30%   19% 15% 15% 22% 19% 20% 15% 17% 15% 15% 15%   40% 44% 50% 40% 44% 43% 35% 35% 40% 45%   5% 5% 5% 5% 5% 10% 10% 5% 10%