APM 236 Second Midterm March 10, 2004 100 points possible

You may not use calculators, cell phones, or PDAs during the exam. Partial credit is possible. Please read the entire test before starting. READ EACH PROBLEM CAREFULLY.

The test ends at 7:00 pm. If even one student does not stop writing when asked, I will not curve the test.

Print your name clearly:

Print your student number clearly:

Please sign here:

Problem 1	out of 20
Problem 2	out of 10
Problem 3	out of 10
Problem 4	out of 10
Problem 5	out of 25
Problem 6	out of 25
Total	out of 100

1. (20 pt) Use the simplex method to solve the following linear programming problem:

$$\begin{gathered}\text{Maximize } -2x + y \\ \text{subject to}\end{gathered}$$

$$\begin{array}{rcrcl} 2x & + & y & \geq & 3 \\ 2x & + & y & \leq & 1 \end{array}$$

$$\text{where } x, y \geq 0$$

For full credit, you have to give an optimal solution and the optimal value. You can't just give the final simplex tableau.

2. (10 pt) Consider the linear programming problem:

$$\text{Maximize } \vec{c}^{\,T}\vec{x}$$
$$\text{subject to}$$
$$A\vec{x} \leq \vec{b}$$
$$\text{where } \vec{x} \geq \vec{0}.$$

Assume that A is an $m \times n$ matrix, $\vec{b} \in \mathbb{R}^m$, and $\vec{c} \in \mathbb{R}^n$.

Prove that if $\vec{b} \geq \vec{0}$ then there will always be feasible solutions. (*That is, show that if* $\vec{b} \geq \vec{0}$ *then you can always find* $\vec{x} \in \mathbb{R}^n$ *such that* $A\vec{x} \leq \vec{b}$ *and* $\vec{x} \geq \vec{0}$.)

3. (10 pt) Consider the following simplex tableau:

	x_1	x_2	x_3	x_4	
x_3	0	-1	1	1	2
x_1	1	0	0	-1	5
	0	-2	0	2	9

This tableau arose in the process of solving a maximization problem. Prove that this maximization problem is unbounded.

4. (10 pt)

a. Consider the following simplex tableau:

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	
x_3	2	0	1	3	-1	1/2	0	1
x_2	1	1	0	-2	1	-3	0	0
x_7	-1	0	0	1	-2	1	1	1
	1	0	0	-6	-2	-2	0	4

This tableau arose as part of a maximization problem. What will you take as your entering variable and as your departing variable? What is the reason for this choice?

b. Consider the following simplex tableau:

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	
x_4	4	0	4	1	1	0	4	0
x_2	2	1	2	0	8	0	-2	0
x_6	6	0	2	0	-6	1	10	3
	9	0	-2	0	-4	0	-6	7

This tableau arose as part of a maximization problem. What will you take as your entering variable and as your departing variable? What is the reason for this choice?

5. (25 pt) Use the simplex method to solve the following linear programming problem:

$$\text{Maximize } z = 2x + y - 3$$
$$\text{subject to}$$

$$\begin{array}{rcrcl} -x & + & y & \leq & 4 \\ -x & + & y & \geq & 1 \\ -x & - & y & \leq & 1 \end{array}$$

$$\text{Where } y \geq 0$$

For full credit, you have to give an optimal solution and the optimal value. You can't just give the final simplex tableau.

6. (25 pt) Use the simplex method to solve the following linear programming problem:

$$\text{Minimize } z = x + 2y + 4$$
$$\text{subject to}$$

$$\begin{array}{rcrcl} -x & + & y & \leq & 4 \\ -x & + & y & \geq & 1 \\ -x & - & y & \leq & 1 \end{array}$$

$$\text{Where } y \geq 0$$

For full credit, you have to give an optimal solution and the optimal value. You can't just give the final simplex tableau.