

You may not use calculators, cell phones, or PDAs during the exam. Partial credit is possible. Please read the entire test over before starting.

The test ends at 7:00 pm. If even one student does not stop writing when asked, I will not curve the test.

Print your name clearly:

Please sign here:

Problem 1	out of 20
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Problem 2	out of 20
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Problem 3	out of 20
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Problem 4	out of 20
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Problem 5	out of 20
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TOTAL	out of 100

1. (20 points) An advertising manager is assigned the task of putting together an advertising campaign for a client. The client has set the objectives of getting at least 160 million exposures, with at least 60 million of those exposures being persons with income of at least \$8000 per year and at least 80 million in the 18 to 40-year-old-age groups. Market studies indicate that an ad in a certain magazine will be seen by 8 million people of whom 3 million will be in the targeted income group and 4 million will be in the targeted age group. Each magazine ad will cost \$40,000. An ad on television will be seen by 40 million people of whom 10 million will be in the targeted income group and 10 million will be in the targeted age group. Each television ad will cost \$200,000. How can the objectives of this advertising campaign be achieved at least cost?

Source: William J. Baumol, "Economic Theory and Operations Analysis", 1972, page 98

Please write this word problem as a linear programming problem.

2. For each of the following four problems, use graphical methods to find the optimal solution(s) if any.

a. (5 points)

$$\begin{array}{ll} \text{Maximize} & x_1 + x_2 \\ \text{subject to} & \\ & x_1 \geq 1 \\ & x_2 \leq 2 \\ & -x_2 \leq 2 \\ & x_1 \geq 0. \end{array}$$

b.

$$\begin{array}{ll} \text{Minimize} & x_1 + x_2 \\ \text{subject to} & \\ & x_1 \geq 1 \\ & x_2 \leq 2 \\ & -x_2 \leq 2 \\ & x_1 \geq 0. \end{array}$$

c.

$$\begin{array}{ll} \text{Minimize} & x_2 \\ \text{subject to} & \\ & x_1 \geq 1 \\ & x_2 \leq 2 \\ & -x_2 \leq 2 \\ & x_1 \geq 0. \end{array}$$

d.

$$\begin{array}{ll} \text{Minimize} & x_2 \\ \text{subject to} & \\ & x_1 \leq 1 \\ & x_2 \leq 0 \\ & -x_2 \leq -2 \\ & x_1 \geq 0. \end{array}$$

- d. (2 points) Write the linear programming problem in canonical form.
- e. (1 points) The feasible solutions of the linear programming problem in part d can be written as “The set of \vec{x} that satisfy $A\vec{x} = \vec{b}$ and $\vec{x} \geq \vec{0}$ ”. What is the matrix A ? What is the vector \vec{b} ?
- f. (3 points) For each extreme point (x_1, x_2) you found in part b, what is the corresponding extreme point of the linear programming problem that you found in part d?

- g. (4 points) Use the first, second, and fourth columns of A to find a basic solution \vec{x} . Is your basic solution a basic feasible solution? Looking at your plot in part a, to what does your basic solution correspond?

4. a. Consider the set of points (x, y) in \mathbb{R}^2 that satisfy the following four inequalities:

$$\begin{cases} x & \geq -2\pi \\ x & \leq 2\pi \\ y & \leq \cos(x) \\ y & \geq -3 \end{cases}$$

(3 points) Plot this set of points. By visual inspection, is this a convex set in \mathbb{R}^2 ?

(7 points) Prove or disprove that this set is convex. *Recall that “a set in \mathbb{R}^2 is convex if (x_0, y_0) and (x_1, y_1) are in the set then $(tx_0 + (1 - t)x_1, ty_0 + (1 - t)y_1)$ is in the set for all $0 < t < 1$.”*

- b. Consider the set of points (x, y) in \mathbb{R}^2 that satisfy the following two inequalities:

$$\begin{cases} y & \geq x^2 \\ y & \leq 1 \end{cases}$$

(2 points) Plot this set of points. By visual inspection, is this a convex set in \mathbb{R}^2 ?

(8 points) Prove or disprove that this set is convex. *Recall that “a set in \mathbb{R}^2 is convex if (x_0, y_0) and (x_1, y_1) are in the set then $(tx_0 + (1 - t)x_1, ty_0 + (1 - t)y_1)$ is in the set for all $0 < t < 1$.”*

5. a. (10 points) Write the following linear programming problem as a linear programming problem in **canonical form**.

Once you have done this, what is your matrix A and your vectors \vec{c} and \vec{b} so that the problem becomes “Maximize $\vec{c} \cdot \vec{x}$ subject to $A\vec{x} = \vec{b}$ and $\vec{x} \geq \vec{0}$?”

$$\begin{array}{rcll} \text{Minimize} & & x_1 - 5x_2 & \\ \text{subject to} & & & \\ -x_1 & + & 4x_2 & = 10 \\ 2x_1 & + & x_2 & \leq 2 \\ x_1 & - & 7x_2 & \geq -3 \\ x_1, x_2 & \geq & 0. & \end{array}$$

- b. (10 points) Write the following linear programming problem in **standard form**.

Once you have done this, what is your matrix A and your vectors \vec{c} and \vec{b} so that the problem becomes “Maximize $\vec{c} \cdot \vec{x}$ subject to $A\vec{x} \leq \vec{b}$ and $\vec{x} \geq \vec{0}$?”

$$\begin{array}{rllll} \text{Maximize} & & -x_1 + 10x_2 & & \\ \text{subject to} & & & & \\ & -x_1 & + & 2x_2 & = & 4 \\ & x_1 & + & 3x_2 & \geq & 6 \\ & x_1 & \geq & 0. & & \end{array}$$