

**You may not use calculators, cell phones, or PDAs during the exam. Partial credit is possible. Please read the entire test over before starting.**

**The test ends at 7:00 pm. If even one student does not stop writing when asked, I will not curve the test.**

Print your name clearly:

Please sign here:

|           |            |
|-----------|------------|
| Problem 1 | out of 20  |
| <hr/>     |            |
| Problem 2 | out of 20  |
| <hr/>     |            |
| Problem 3 | out of 20  |
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| Problem 4 | out of 20  |
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| Problem 5 | out of 20  |
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| TOTAL     | out of 100 |

1. a. (10 points) Write the following linear programming problem as a linear programming problem in **canonical form**.

Once you have done this, what is your matrix  $A$  and your vectors  $\vec{c}$  and  $\vec{b}$  so that the problem becomes “Maximize  $\vec{c} \cdot \vec{x}$  subject to  $A\vec{x} = \vec{b}$  and  $\vec{x} \geq \vec{0}$ ?”

$$\begin{array}{ll} \text{Minimize} & -x_1 + 2x_2 \\ \text{subject to} & \\ & x_1 + x_2 \leq 3 \\ & 2x_1 - 3x_2 \geq 7 \\ & x_1 - x_2 = 8 \\ & x_1, x_2 \geq 0. \end{array}$$

- b. (10 points) Write the following linear programming problem in **standard form**.

Once you have done this, what is your matrix  $A$  and your vectors  $\vec{c}$  and  $\vec{b}$  so that the problem becomes “Maximize  $\vec{c} \cdot \vec{x}$  subject to  $A\vec{x} \leq \vec{b}$  and  $\vec{x} \geq \vec{0}$ ?”

$$\begin{array}{llll} \text{Maximize} & & -2x_1 + 7x_2 & \\ \text{subject to} & & & \\ & x_1 & - & 2x_2 & \geq & 9 \\ & x_1 & + & x_2 & = & 3 \\ & x_2 & \geq & 0. & & \end{array}$$



- d. (2 points) Write the linear programming problem in canonical form.
- e. (1 points) The feasible solutions of the linear programming problem in part d can be written as “The set of  $\vec{x}$  that satisfy  $A\vec{x} = \vec{b}$  and  $\vec{x} \geq \vec{0}$ ”. What is the matrix  $A$ ? What is the vector  $\vec{b}$ ?
- f. (3 points) For each extreme point  $(x_1, x_2)$  you found in part b, what is the corresponding extreme point of the linear programming problem that you found in part d?

- g. (4 points) Use the first, second, and fourth columns of  $A$  to find a basic solution  $\vec{x}$ . Is your basic solution a basic feasible solution? Looking at your plot in part a, to what does your basic solution correspond?

3. a. Consider the set of points  $(x, y)$  in  $\mathbb{R}^2$  that satisfy the following four inequalities:

$$\begin{cases} x & \geq 0 \\ x & \leq 4\pi \\ y & \geq \sin(x) \\ y & \leq 2 \end{cases}$$

(3 points) Plot this set of points. By visual inspection, is this a convex set in  $\mathbb{R}^2$ ?

(7 points) Prove or disprove that this set is convex. *Recall that “a set in  $\mathbb{R}^2$  is convex if  $(x_0, y_0)$  and  $(x_1, y_1)$  are in the set then  $(tx_0 + (1 - t)x_1, ty_0 + (1 - t)y_1)$  is in the set for all  $0 < t < 1$ .”*

b. Consider the set of points  $(x, y)$  in  $\mathbb{R}^2$  that satisfy the following two inequalities:

$$\begin{cases} y \geq x^2 \\ y \leq 1 \end{cases}$$

(2 points) Plot this set of points. By visual inspection, is this a convex set in  $\mathbb{R}^2$ ?

(8 points) Prove or disprove that this set is convex. *Recall that “a set in  $\mathbb{R}^2$  is convex if  $(x_0, y_0)$  and  $(x_1, y_1)$  are in the set then  $(tx_0 + (1 - t)x_1, ty_0 + (1 - t)y_1)$  is in the set for all  $0 < t < 1$ .”*

4. (20 points) A welfare recipient decides to see if his knowledge of linear programming can help him with his financial problems. Peanut butter costs 20 cents an ounce and contains 1 unit of carbohydrate and 1 unit of protein. A small loaf of bread costs 12 cents and contains 1 unit of carbohydrate and no protein. A cup of milk costs 16 cents and contains 1 unit of protein and no carbohydrate. He estimates that he needs .9 units of carbohydrate and .6 units of protein per day. How can he get at least that much nutrient from these foods at least cost?

*Source: Harold W. Kuhn, unpublished class notes.*

Please write this word problem as a linear programming problem.

5. For each of the following four problems, use graphical methods to find the optimal solution(s) if any.

a. (5 points)

$$\begin{array}{ll} \text{Maximize} & x_1 + x_2 \\ \text{subject to} & \\ & -x_2 \leq 1 \\ & x_1 \leq 2 \\ & -x_1 \leq -1 \\ & x_1 \geq 0. \end{array}$$

b. (5 points)

$$\begin{array}{ll} \text{Minimize} & x_1 + x_2 \\ \text{subject to} & \\ & -x_2 \leq 1 \\ & x_1 \leq 2 \\ & -x_1 \leq -1 \\ & x_1 \geq 0. \end{array}$$

c. (5 points)

$$\begin{array}{ll} \text{Maximize} & x_1 \\ \text{subject to} & \\ & -x_2 \leq 1 \\ & x_1 \leq 2 \\ & -x_1 \leq -1 \\ & x_1 \geq 0. \end{array}$$

d. (5 points)

$$\begin{array}{ll} \text{Maximize} & x_1 + 2x_2 \\ \text{subject to} & \\ & -x_2 \leq 1 \\ & x_1 \leq 1 \\ & -x_1 \leq -2 \\ & x_1 \geq 0. \end{array}$$