# Practice problems from game theory

1. Find the von Neummann value and the optimal strategy for each player in the matrix game with pay-off matrix

$$\begin{pmatrix} -1 & 0 & 2 & -2 & 0 \\ 1 & -2 & -4 & 2 & 2 \\ 0 & -1 & 1 & 1 & -1 \\ 0 & 5 & 4 & 2 & 0 \end{pmatrix}$$

2. Find the von Neumann value and the optimal strategy for each player in the following game.

   Player I and player II each have a penny and a nickel. They each choose one of their coins and display them simultaneously. If the coins are the same, player I wins the sum of the coins from player II; if the coins are different, player II wins a nickel from player I.

3. Find the von Neumann value and the optimal strategy for each player in the following game.

   Player I has the two of spades (2♠) and the three of hearts (3♡) from a deck of (standard) playing cards and player II has the three of spades (3♠) and the four of hearts (4♡). They each choose one of their cards and display them simultaneously. If the colors are the same, player I wins; if the colors are different, player II wins. If player I plays the 2♠, the payoff consists of the difference of the numbers on the cards; if player I plays the 3♡, the payoff consists of the sum of the numbers on the cards.

4. Consider the game below:

   Player I and player II each have two pennies. Each player holds 0, 1, or 2 pennies in her left hand and the remainder of the pennies in her right hand. Each player reveals both hands simultaneously. If the number of coins in one of player I's hands is greater than the number of coins in the respective hand of player II, player I wins the difference in pennies; otherwise no money is exchanged.

   a) Which player is favored in the game? (No work is required here!)

   b) Find the von Neumann value and the optimal strategy for each player in the game. If any player has inifinitely many optimal strategies, find all optimal strategies.

   c) If player II owed $100 to player I, approximately how many rounds of the game would have to be played, on the average, to cancel the debt?

   d) Assume that player II is allowed three pennies. Repeat parts a, b, and c for this new game.

5. Find the von Neumann value and the optimal strategy for each player in the game below:

   Player I and player II are each dealt a single card face down from a deck of three playing cards, a jack (J), a queen (Q), and a king (K). The ranking of these cards, from lowest to highest, is J, Q, and K; the suits of these cards is irrelevant. Each player looks at her card. Player I now has two options:

   **PASS**: Both cards are revealed and the high card wins. If player I has the high card, she wins $2 from player II. If player I has the low card, she pays $3 to player II.

   **BET**: Player I puts $1 into the pot.

   If player I bets, player II now has two options:

   **PASS**: Player II adds $3 to the pot and player I adds $1 to the pot. Both cards are revealed and the high hand wins the pot.

**SEE**: Player II puts \$1 in the pot.

In the event that player II sees, both cards are revealed and the high hand wins the pot. The cards are then returned to the deck.

6. For more problems, see chapter 5 of J.K. Strayer's "Linear Programming and its Applications" and chapters 1 and 2 of M. Drescher's "The Mathematics of Games of Strategy". I've put both books on reserve in the MathStat library in the basement of Sidney Smith Hall. *Note: Strayer uses Tucker tableaux rather than Dantzig tableau in the simplex method, so it's a little confusing to translate his tableau to the ones you're used to. Also, Drescher presents a way to solve these problems which is basically the simplex method in disguise. Don't learn Drescher or Strayer's versions of the simplex method, just look at them for games and how to construct payoff matrices. Once you've constructed the payoff matrix, you can do your usual method of removing the dominating columns and rows and then applying the simplex method.*