Kolman + Beck pp 338-339 # 1-7

#1.)

$$C = \begin{pmatrix} 4 & 2 & 3 & 5 \\ 2 & 3 & 4 & 6 \\ 3 & 2 & 5 & 2 \\ 2 & 5 & 3 & 4 \end{pmatrix} \rightarrow C' = \begin{pmatrix} 2 & 0 & 0 & 3 \\ 0 & 1 & 1 & 4 \\ 1 & 0 & 2 & 0 \\ 0 & 3 & 0 & 2 \end{pmatrix}$$

first did rows, then cols.

$$\begin{pmatrix} 2 & 0^* & 0 & 3 \\ 0^* & 1 & 1 & 4 \\ 1 & 0 & 2 & 0^* \\ 0 & 3 & 0^* & 2 \end{pmatrix}$$

is a valid assignment.
Cool! Finished at step 2 of the Hungarian algorithm

cost = 2 + 2 + 2 + 3 = 9

#2)

$$C = \begin{pmatrix} 3 & 2 & 5 & 8 & 9 \\ 6 & 7 & 4 & 2 & 3 \\ 5 & 3 & 5 & 4 & 2 \\ 4 & 7 & 3 & 2 & 4 \\ 2 & 6 & 5 & 5 & 3 \end{pmatrix} \rightarrow C' = \begin{pmatrix} 1 & 0^* & 2 & 6 & 7 \\ 4 & 5 & 1 & 0^* & 1 \\ 3 & 1 & 2 & 2 & 0^* \\ 2 & 5 & 0^* & 0 & 2 \\ 0^* & 4 & 2 & 3 & 1 \end{pmatrix}$$

is a valid assignment.
Again, finished at step 2

Cost of solution = 2 + 2 + 2 + 3 + 2 = 11

#3

$$C = \begin{pmatrix} 3 & 4 & 0 & 2 & 6 & 7 \\ 4 & 6 & 4 & 5 & 3 & 6 \\ 5 & 7 & 7 & 8 & 2 & 8 \\ 0 & 8 & 8 & 4 & 6 & 4 \\ 6 & 4 & 3 & 7 & 4 & 9 \\ 7 & 5 & 5 & 0 & 6 & 7 \end{pmatrix} \rightarrow C' = \begin{pmatrix} 3 & 3 & 0^* & 2 & 6 & 4 \\ 1 & 2 & 1 & 2 & 0^* & 0 \\ 3 & 4 & 5 & 6 & 0 & 3 \\ 0^* & 7 & 8 & 4 & 6 & 1 \\ 3 & 0^* & 0 & 4 & 1 & 3 \\ 7 & 4 & 5 & 0^* & 6 & 4 \end{pmatrix}$$

is not a valid assignment. So we go to step 3 of the Hungarian algorithm to fix row 3

0 in (3,5) to
0* in (2,5) to
0 in (2,6)

and stop since (2,6) is unassigned, we can flip!

$$\begin{pmatrix} 3 & 3 & 0^* & 2 & 6 & 4 \\ 1 & 2 & 1 & 2 & 0 & 0^* \\ 3 & 4 & 5 & 6 & 0^* & 3 \\ 0^* & 7 & 8 & 4 & 6 & 1 \\ 3 & 0^* & 0 & 4 & 1 & 3 \\ 7 & 4 & 5 & 0^* & 6 & 4 \end{pmatrix}$$

is a valid assignment!

$$\text{cost} = 0 + 6 + 2 + 0 + 4 + 0$$
$$= 12$$

#4

$$C = \begin{pmatrix} 3 & 2 & 7 & 4 & 8 \\ 5 & 4 & 3 & 8 & 5 \\ 3 & 7 & 9 & 1 & 2 \\ 4 & 2 & 6 & 5 & 7 \\ 2 & 8 & 4 & 6 & 6 \end{pmatrix} \longrightarrow C' = \begin{pmatrix} 1 & 0^* & 5 & 2 & 5 \\ 2 & 1 & 0^* & 5 & 1 \\ 2 & 6 & 8 & 0^* & 0 \\ 2 & 0 & 4 & 3 & 4 \\ 0^* & 6 & 2 & 4 & 3 \end{pmatrix}$$

is not a valid assignment. Got to fix row 4

0 in $(4,2)$ to

0^* in $(1,2)$

no 0 in row 1 $\Rightarrow$ column 2 is necessary.
delete the last two cells from my chain and see if there was some other 0 I could have chosen But since the chain only has two cells, I'm done and now must choose necessary rows. i.e. find rows that have 0^* in columns other than 2

$\Rightarrow$ rows 2, 3, 5 are necessary

$$\begin{pmatrix} 1 & 0 & 5 & 2 & 5 \\ 2 & 1 & 0 & 5 & 1 \\ 2 & 6 & 8 & 0 & 0 \\ 2 & 0 & 4 & 3 & 4 \\ 0 & 6 & 2 & 4 & 3 \end{pmatrix}$$

subtract 1 from uncovered cells and add 1 to doubly covered ones.

$$\begin{pmatrix} 0^* & 0 & 4 & 1 & 4 \\ 2 & 2 & 0^* & 5 & 1 \\ 2 & 7 & 8 & 0^* & 0 \\ 1 & 0^* & 4 & 3 & 4 \\ 0 & 7 & 2 & 4 & 3 \end{pmatrix}$$

is the new matrix. I assign 0's as before... (step 2)

is not a valid assignment. So I move to step 3 to fix row 5.

0 in $(5,1)$ to
0^* in $(1,1)$ to
0 in $(1,2)$ to
0^* in $(4,2)$

⇒ column 2 is necessary
I remove the last 2 cells and am at

0 in $(5,1)$ to
0^* in $(1,1)$

there were no other 0's I could have connected to (other than (1,2)) so I can't extend to a different and longer path. ⇒ remove two more cells (and declare column 1 necessary). But now I've run out of path!

⇒ columns 1 & 2 are necessary.

⇒ rows 2 and 3 are necessary.

$$\begin{pmatrix} 0 & 0 & 4 & 1 & 4 \\ 2 & 2 & 0 & 5 & 1 \\ 2 & 7 & 8 & 0 & 0 \\ 1 & 0 & 4 & 3 & 4 \\ 0 & 7 & 2 & 4 & 3 \end{pmatrix}$$

subtract 1 from uncovered, add 1 to doubly covered.

and go to step 2

$$\begin{pmatrix} 0^* & 0 & 3 & 0 & 3 \\ 3 & 3 & 0^* & 5 & 1 \\ 3 & 8 & 8 & 0^* & 0 \\ 1 & 0^* & 3 & 2 & 3 \\ 0 & 7 & 1 & 3 & 2 \end{pmatrix}$$

again this is not a valid assignment. ⇒ go to step 3 to fix row 5

0 in $(5,1)$

0^* in $(1,1)$

0 in $(1,2)$

0^* in $(4,2)$

and stop declare column 2 necessary and trim the last two cells.

⇒ 0 in (5,1) to
0^* in (1,1)

Now I look to see if there are other unassigned 0's in row 1. There are! So I continue my path

0 in (5,1) to
0^* in (1,1) to
0 in (1,4) to
0^* in (3,4) to
0 in (3,5)

I terminated at 0 ⇒ can flip and increase the # of 0^*s.

$$\begin{pmatrix} 0 & 0 & 3 & 0^* & 3 \\ 3 & 3 & 0^* & 5 & 1 \\ 3 & 8 & 8 & 0 & 0^* \\ 1 & 0^* & 3 & 2 & 3 \\ 0^* & 7 & 1 & 3 & 2 \end{pmatrix}$$

is a valid assignment.

cost = 4 + 3 + 2 + 2 + 2
= 13

#5

$$C = \begin{pmatrix} 3 & 5 & 4 & 2 & 8 & 1 \\ 8 & 3 & 6 & 6 & 4 & 3 \\ 4 & 4 & 8 & 8 & 3 & 5 \\ 3 & 8 & 7 & 4 & 9 & 7 \\ 7 & 7 & 9 & 2 & 3 & 5 \\ 9 & 7 & 2 & 7 & 5 & 8 \end{pmatrix} \longrightarrow C' = \begin{pmatrix} 2 & 4 & 3 & 1 & 7 & 0^* \\ 5 & 0^* & 3 & 3 & 1 & 0 \\ 1 & 1 & 5 & 5 & 0^* & 2 \\ 0^* & 5 & 4 & 1 & 6 & 4 \\ 5 & 5 & 7 & 0^* & 1 & 3 \\ 7 & 5 & 0^* & 5 & 3 & 6 \end{pmatrix}$$

and terminates at step 2.

Cost $= 1 + 3 + 3 + 3 + 2 + 2 = 14$

#6

$$C = \begin{pmatrix} 9 & 7 & 4 & 7 & 3 \\ 0 & 8 & 3 & 5 & 8 \\ 6 & 3 & 2 & 8 & 9 \\ 5 & 6 & 7 & 0 & 4 \\ 2 & 9 & 5 & 7 & 3 \end{pmatrix} \longrightarrow C' = \begin{pmatrix} 6 & 3 & 1 & 4 & 0^* \\ 0^* & 7 & 3 & 5 & 8 \\ 4 & 0^* & 0 & 6 & 7 \\ 5 & 5 & 7 & 0^* & 4 \\ 0 & 6 & 3 & 5 & 1 \end{pmatrix}$$

is not a valid assignment.
go to step 2.

0 in $(5,1)$ to
0^* in $(2,1)$

⇒ column 1 is necessary

⇒ rows 1, 3, 3 4 are necessary.

⇒

6	3	1	4	0
0	7	3	5	8
4	0	0	6	7
5	5	7	0	4
0	6	3	5	1

subtract 1 from uncovered, add 1 to doubly covered, go to step 2.

7	3	1	4	0*
0*	6	2	4	7
5	0*	0	6	7
6	5	7	0*	4
0	5	2	4	0*

is not a valid assignment

first try 0 in (5,1) to 0* in (2,1)

⇒ column 1 is necessary. delete the last 2 cells. ⇒ are back to where we started.

Try another 0 in row 5: 0 in (5,5) to 0* in (1,5)

⇒ column 5 is necessary. delete last two cells ⇒ back to where we started.

⇒ necessary columns are 1 & 5.

⇒ necessary rows are 3 & 4. ready for step 4!

$$\begin{pmatrix} 7 & 3 & 1 & 4 & 0 \\ 0 & 6 & 2 & 4 & 7 \\ 5 & 0 & 0 & 6 & 7 \\ 6 & 5 & 7 & 0 & 4 \\ 0 & 5 & 2 & 4 & 0 \end{pmatrix}$$

subtract 1 from uncovered, add 1 to doubly covered, go to step 2.

$$\begin{pmatrix} 7 & 2 & 0^* & 3 & 0 \\ 0^* & 5 & 1 & 3 & 7 \\ 6 & 0^* & 0 & 6 & 8 \\ 7 & 5 & 7 & 0^* & 5 \\ 0 & 4 & 1 & 3 & 0^* \end{pmatrix}$$

is a valid assignment!

cost $= 4 + 0 + 3 + 0 + 3 = 10$

7 desks in rooms

702, 705, 708, 709, 713

go into rooms

706 707 712 714 715

transport cost = distance moved.

i.e. cost of moving desk from 702 to 706 is 4.

$$C = \begin{pmatrix} 4 & 5 & 10 & 12 & 13 \\ 1 & 2 & 7 & 9 & 10 \\ 2 & 1 & 4 & 6 & 7 \\ 3 & 2 & 3 & 5 & 6 \\ 7 & 6 & 1 & 1 & 2 \end{pmatrix} \to C' = \begin{pmatrix} 0^* & 1 & 6 & 8 & 8 \\ 0 & 1 & 6 & 8 & 8 \\ 1 & 0^* & 3 & 5 & 5 \\ 1 & 0 & 1 & 3 & 3 \\ 6 & 5 & 0^* & 0 & 0 \end{pmatrix}$$

applying step 3 to fix row 2, we see that column 1 is necessary.

applying step 3 to fix row 4, we see that column 2 is necessary

$\Rightarrow$ row 5 is necessary

$$\begin{matrix} 0 & 1 & 6 & 8 & 8 \\ 0 & 1 & 6 & 8 & 8 \\ 1 & 0 & 3 & 5 & 5 \\ 1 & 0 & 1 & 3 & 3 \\ 6 & 5 & 0 & 0 & 0 \end{matrix}$$

subtract 1 from uncovered entries, add 1 to doubly covered entries, apply step 2.

$$\begin{pmatrix} 0^* & 1 & 5 & 7 & 7 \\ 0 & 1 & 5 & 7 & 7 \\ 1 & 0^* & 2 & 4 & 4 \\ 1 & 0 & 0^* & 2 & 2 \\ 7 & 6 & 0 & 0^* & 0 \end{pmatrix}$$

not a valid assignment.

applying step 3 to row 1, column 1 is necessary ⇒ rows 3, 4, 5 are necessary

$$\begin{pmatrix} 0 & 1 & 5 & 7 & 7 \\ 0 & 1 & 5 & 7 & 7 \\ 1 & 0 & 2 & 4 & 4 \\ 1 & 0 & 0 & 2 & 2 \\ 7 & 6 & 0 & 0 & 0 \end{pmatrix}$$

subtract 1 from uncovered, add 1 to doubly covered, apply step 2

$$\begin{pmatrix} 0^* & 0 & 4 & 6 & 6 \\ 0 & 0^* & 4 & 6 & 6 \\ 2 & 0 & 2 & 4 & 4 \\ 2 & 0 & 0^* & 2 & 2 \\ 8 & 6 & 0 & 0^* & 0 \end{pmatrix}$$ is not a valid assignment.

apply step 3 to try and fix row 3.

0 in $(3,2)$ to

0^* in $(2,2)$ to

0 in $(2,1)$ to

0^* in $(1,1)$ to

$\Rightarrow$ column 1 necessary

$\Rightarrow$ column 2 necessary

$\Rightarrow$ rows 4 & 5 necessary.

$$\begin{pmatrix} 0 & 0 & 4 & 6 & 6 \\ 0 & 0 & 4 & 6 & 6 \\ 2 & 0 & 2 & 4 & 4 \\ 2 & 0 & 0 & 2 & 2 \\ 8 & 6 & 0 & 6 & 0 \end{pmatrix}$$

subtract 2 from uncovered,
add 2 to doubly covered,
apply step 2.

$$\begin{pmatrix} 0^* & 0 & 2 & 4 & 4 \\ 0 & 0^* & 2 & 4 & 4 \\ 2 & 0 & 0^* & 2 & 2 \\ 4 & 2 & 0 & 2 & 2 \\ 10 & 8 & 0 & 0^* & 0 \end{pmatrix}$$

is not a valid assignment :(

0 in (4,3) to
0^* in (3,3) to
0 in (3,2) to
0^* in (2,2) to
0 in (2,1) to
0^* in (1,1)

$\Rightarrow$ columns 1, 2, 3 are necessary

$\Rightarrow$ row 5 is necessary

$$\begin{pmatrix} 0 & 0 & 2 & 4 & 4 \\ 0 & 0 & 2 & 4 & 4 \\ 2 & 0 & 0 & 2 & 2 \\ 4 & 2 & 0 & 2 & 2 \\ 10 & 8 & 0 & 0 & 0 \end{pmatrix}$$

subtract 2 from uncovered. add 2 to doubly covered, go to step 2

$$\begin{pmatrix} 0^* & 0 & 2 & 2 & 2 \\ 0 & 0^* & 2 & 2 & 2 \\ 2 & 0 & 0^* & 0 & 0 \\ 4 & 2 & 0 & 0^* & 0 \\ 12 & 10 & 2 & 0 & 0^* \end{pmatrix}$$

done! (they could have had new desks built in the time that took me...)

cost = 4 + 2 + 4 + 5 + 2 = 17

desk 702 → room 706
desk 705 → room 707
desk 708 → room 712
desk 709 → room 714
desk 713 → room 715