Kolman + Beck p 325 #8, #10, #11, #12

#8) First I apply Vogel's method to find a cheap initial basic feasible solution

					Supply
	2	5	6	3	100
	9	6	2	1	90
	7	7	2	4	130
demand	70	50	30	120	

supply = demand? $100+90+130 = 320$ ← supply

$70+50+30+120 = 270$ ← demand

Since demand < supply, I deal with this by introducing a "dummy warehouse". i.e a 5th warehouse whose demand is 50. The costs of shipping to this warehouse will be zero (i.e. the "warehouse" is just a back room at the factory itself.)

					supply	
2	5	6	3	0	100	<2>
9	6	2	1	0	90	<1>
7	7	2	4	0	130	<2>
70	50	30	120	50		
<5>	<1>	<4>	<2>	<0>		

demand 70 50 30 120 50

differences: <5> <1> <4> <2> <0>

Column 1 has largest difference ... let's fill it first

2 70	5	6	3	0	30	<3>
9	6	2	1	0	90	<1>
7	7	2	4	0	130	<2>
0	50	30	120	50		
	<1>	<4>	<2>	<0>		

oops! a mistake! it's really <0>.

Column 3 has the largest difference. let's fill it in now

see note at end

2 70	5	6	3	0	30	<3>
9	6	2	1	0	90	<1>
7	7	2 30	4	0	100	<4>
0	50	0	120	50		
	<1>		<2>	<0>		

row 3 has the largest difference let's fill it in now

$\boxed{2}$ 70	$\boxed{5}$	$\boxed{6}$	$\boxed{3}$	$\boxed{0}$	30	<3>
$\boxed{9}$	$\boxed{6}$	$\boxed{2}$	$\boxed{1}$	$\boxed{0}$	90	<1>
$\boxed{7}$	$\boxed{7}$	$\boxed{2}$ 30	$\boxed{4}$ 50	$\boxed{0}$ 50	$\cancel{0}$	
0	50	0	70	0		
	<1>		<2>			

row 1 has the largest difference. Let's fill it in now

$\boxed{2}$ 70	$\boxed{5}$	$\boxed{6}$	$\boxed{3}$ 30	$\boxed{0}$ 0	$\cancel{0}$	
$\boxed{9}$	$\boxed{6}$	$\boxed{2}$	$\boxed{1}$	$\boxed{0}$	90	<1>
$\boxed{7}$	$\boxed{7}$	$\boxed{2}$ 30	$\boxed{4}$ 50	$\boxed{0}$ 50	0	
0	50	0	40	0		
	<1>		<3>			

fill in column 4 now

$\boxed{2}$ 70	$\boxed{5}$	$\boxed{6}$	$\boxed{3}$ 30	$\boxed{0}$	0
$\boxed{9}$	$\boxed{6}$	$\boxed{2}$	$\boxed{1}$ 40	$\boxed{0}$	50
$\boxed{7}$	$\boxed{7}$	$\boxed{2}$ 30	$\boxed{4}$ 50	$\boxed{0}$ 50	0
0	50	0	0	0	

done w/ initial guess: put 50 into (2,2) slot

Oof! Vogel's method is done.

2) 70	5)	6)	3) 30	0)	100
9)	6) 50	2)	1) 40	0)	90
7)	7)	2) 30	4) 50	0) 50	130
70	50	30	120	50	

Cost = $830

basic variables:

$X_{11}, X_{14}, X_{22}, X_{24}, X_{33}, X_{34}, X_{35}$

15 unknowns. $(3 \cdot 5)$

7 constraining equations $(3+5-1)$

We now solve the dual system:

$$
\begin{aligned}
V_1 + W_1 &= 2 \\
V_1 + W_4 &= 3 \\
V_2 + W_2 &= 6 \\
V_2 + W_4 &= 1 \\
V_3 + W_3 &= 2 \\
V_3 + W_4 &= 4 \\
V_3 + W_5 &= 0
\end{aligned}
$$

7 eqns, 8 unknowns $\Rightarrow$ one degree of freedom, as expected. Let's take $V_1 = 0$

$v_1 = 0$ $\qquad$ $w_1 = 2$

$v_2 = -2$ $\qquad$ $w_2 = 8$

$v_3 = 1$ $\qquad$ $w_3 = 1$

$w_4 = 3$

$w_5 = -1$

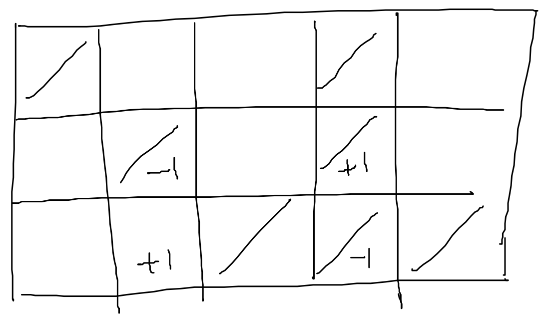
Compute the objective row entries for the 8 nonbasic variables

$$(obj)_{12} = 0 + 8 - 5 = 3$$
$$(obj)_{13} = 0 + 1 - 6 = -5$$
$$(obj)_{15} = 0 - 1 - 0 = -1$$
$$(obj)_{21} = -2 + 2 - 9 = -9$$
$$(obj)_{23} = -2 + 1 - 2 = -3$$
$$(obj)_{26} = -2 - 1 - 0 = -3$$
$$(obj)_{31} = 1 + 2 - 7 = -4$$
$$(obj)_{32} = 1 + 8 - 7 = 2$$

$\Rightarrow$ x_{12} and x_{32} are candidate entering variables.

	+1		−1	
	−1		+1	

if x_{12} enters, x_{14} will depart and $30 \cdot 3 = 90$ \$ will be saved.



if x_{32} enters, x_{22} will depart* and we'll save $50 \cdot 2 = 100\$$

So it's better to have x_{32} enter!

* Note: both x_{22} and x_{34} will go to zero simultaneously. I'll keep one of them around as a basic variable even though it equals zero. (degenerate case).

[2] 70	[5]	[6]	[3] 30	[0]	100
[9]	[6]	[2]	[1] 90	[0]	90
[7]	[7] 50	[2] 30	[4] 0	[0] 50	130
70	50	30	120	50	

$\boxed{\text{Cost} = \$730}$

We again go to the dual problem using our new set of basic variables: $x_{11}, x_{14}, x_{24}, x_{32}, x_{33}, x_{34}, x_{35}$

$v_1 + w_1 = 2$

$v_1 + w_4 = 3$

$v_2 + w_4 = 1$

$v_3 + w_2 = 7$ $\quad$ $v_3 + w_4 = 4$

$v_3 + w_3 = 2$ $\quad$ $v_3 + w_5 = 0$

Solving, we find

$v_1 = 0$ $\qquad$ $w_1 = 2$

$v_2 = -2$ $\qquad$ $w_2 = 6$

$v_3 = 1$ $\qquad$ $w_3 = 1$

$w_4 = 3$

$w_5 = -1$

Compute the objective row for the 8 nonbasic vars:

$(obj)_{12} = 0 + 6 - 5 = 1$ ←

$(obj)_{13} = 0 + 1 - 6 = -5$

$(obj)_{15} = 0 - 1 - 0 = -1$

$(obj)_{21} = -2 + 2 - 9 = -9$

$(obj)_{22} = -2 + 6 - 6 = -2$

$(obj)_{23} = -2 + 1 - 2 = -3$

$(obj)_{25} = -2 - 1 - 0 = -3$

$(obj)_{31} = 1 + 2 - 7 = -4$

Take X_{12} entering ⇒

/	+1		/ -1	
			/	
	/ -1	/	/ +1	/

X_{14} departs
cost decreases
by \$30

our new tableau

$\boxed{\text{cost} = \$700}$

[2] 70	[5] 30	[6]	[3]	[0]	100
[9]	[6]	[2]	[1] 90	[0]	90
[7]	[7] 20	[2] 30	[4] 30	[0] 50	130
70	50	30	120	50	

we again go to the dual problem, using our new set of basic variables: $X_{11}, X_{12}, X_{24}, X_{32}, X_{33}, X_{34}, X_{35}$

$V_1 + W_1 = 2$

$V_1 + W_2 = 5 \qquad V_3 + W_3 = 2$

$V_2 + W_4 = 1 \qquad V_3 + W_4 = 4$

$V_3 + W_2 = 7 \qquad V_3 + W_5 = 0$

$\Rightarrow \quad V_1 = 0 \qquad W_1 = 2$

$V_2 = -1 \qquad W_2 = 5$

$V_3 = 2 \qquad W_3 = 0$

$W_4 = 2$

$W_5 = -2$

$\Rightarrow \quad (obj)_{13} = 0 + 0 - 6 < 0 \qquad (obj)_{21} = -1 + 2 - 9 < 0 \qquad (obj)_{31} = 2 + 2 - 7 < 0$

$(obj)_{14} = 0 + 2 - 3 < 0 \qquad (obj)_{22} = -1 + 5 - 6 < 0$

$(obj)_{15} = 0 - 2 - 0 < 0 \qquad (obj)_{23} = -1 + 0 - 2 < 0$

$(obj)_{25} = -1 - 2 - 0 < 0$

all < 0

$\Rightarrow$ we have found an opt. soln!

Notice that I made a mistake in my Vogel's method early on. But in any case, I did find an initial basic feasible solution and the rest of the stuff is right.

#10.

4	3	2	5	6	70	<1>
8	3	4	5	7	80	<1>
6	8	6	7	5	60	<1>
4	3	5	2	4	120	<1>
60	50	50	70	100		
<0>	<0>	<2>	<3>	<1>		

supply = 70 + 80 + 60 + 120 = 330

demand = 60 + 50 + 50 + 70 + 100 = 330

this time, I'll do Vogel's method right :)

Fill column 4 first

4	3	2	5	6	70	<1>
8	3	4	5	7	80	<1>
6	8	6	7	5	60	<1>
4	3	5	2 70	4	50	<1>
60	50	50	0	100		
<0>	<0>	<2>		<1>		

Fill column 3 now

4	7	2 50	5	6	20	<1>
8	3	4	5	7	80	<4>
6	8	6	7	5	60	<1>
4	3	5	2 70	4	60	<1>
60	50	0	0	100		
<0>	<0>			<1>		

Fill row 2 now

4	3	2 50	5	6	20	<2>
8	3 50	4	5	7 30	0	
6	8	6	7	5	60	<1>
4	3	5	2 70	4	50	<0>
60	0	0	0	70		
<0>				<1>		

fill row 1 now

4 20	3	2 50	5	6	0	
8	3 50	4	5	7 30	0	
6	8	6	7	6	60	<1>
4	3	5	2 70	4	50	<0>
40	0	0	0	70		
<2>				<1>		

Fill column 1 now. This then determines the rest of the tableau.

[4] 20	[3]	[2] 50	[5]	[6]	70
[8]	[3] 50	[4]	[5]	[7] 30	80
[6]	[8]	[6]	[7]	[5] 60	60
[4] 40	[3]	[6]	[2] 70	[4] 10	120
60	50	50	70	100	

cost = $1180

basic variables:

x_{11}: $v_1 + w_1 = 4$

x_{13}: $v_1 + w_3 = 2$

x_{22}: $v_2 + w_2 = 3$

x_{25}: $v_2 + w_5 = 7$

x_{35}: $v_3 + w_5 = 5$

x_{41}: $v_4 + w_1 = 4$

x_{44}: $v_4 + w_4 = 2$

x_{45}: $v_4 + w_5 = 4$

$\Rightarrow$

$v_1 = 0$, $v_2 = 3$, $v_3 = 1$, $v_4 = 0$

$w_1 = 4$, $w_2 = 0$, $w_3 = 2$, $w_4 = 2$, $w_5 = 4$

I'll use the book's cartoon for filling in the objective row (tableaux 5.21, 5.22)

4 20	3 -3	2 50	5 -3	4 -2	70	0
8 -1	3 50	4 1	5 0	7 30	80	3
6 -1	8 -7	6 -3	7 -4	5 60	60	1
4 40	3 -3	5 -3	2 70	4 10	120	0
60	50	50	70	100		
4	0	2	2	4		

We see that x_{23} is a choice of entering variable that will lead to a cost savings.

+1		-1		
		+1		-1
-1				+1

We see x_{25} will depart and $30 will be saved.

new tableau:

4 50	3	2 20	5	6
8	3 50	4 30	5	7
6	8	6	7	5 60
4 10	3	5	2 70	4 40

Cost = $1150

the dual problem:

$v_1 + w_1 = 4$

$v_1 + w_3 = 2$

$v_2 + w_2 = 3$

$v_2 + w_3 = 4$

$v_3 + w_5 = 5$

$v_4 + w_1 = 4$

$v_4 + w_4 = 2$

$v_4 + w_5 = 4$

$\rightarrow$

$v_1 = 0$ $w_1 = 4$

$v_2 = 2$ $w_2 = 1$

$v_3 = 1$ $w_3 = 2$

$v_4 = 0$ $w_4 = 2$

$w_5 = 4$

$\boxed{4}$ 60	$\boxed{3}$ -2	$\boxed{2}$ 20	$\boxed{5}$ -3	$\boxed{6}$ -2	70	0
$\boxed{8}$ -2	$\boxed{3}$ 50	$\boxed{4}$ 30	$\boxed{6}$ -1	$\boxed{7}$ -1	80	2
$\boxed{6}$ -1	$\boxed{8}$ -6	$\boxed{6}$ -3	$\boxed{7}$ -4	$\boxed{5}$ 60	60	1
$\boxed{4}$ 10	$\boxed{3}$ -2	$\boxed{5}$ -3	$\boxed{2}$ 70	$\boxed{4}$ 40	120	0
60	50	50	70	100		
4	1	2	2	4		

the objective row is negative $\Rightarrow$ we have an optimal solution !!

#11

$\boxed{4}$	$\boxed{2}$	$\boxed{9}$	$\boxed{7}$	1
$\boxed{7}$	$\boxed{8}$	$\boxed{5}$	$\boxed{6}$	1
$\boxed{3}$	$\boxed{3}$	$\boxed{4}$	$\boxed{1}$	1
$\boxed{7}$	$\boxed{5}$	$\boxed{2}$	$\boxed{6}$	1
1	1	1	1	

supply = demand = 4 Apply Vogel's method, I find

$\boxed{4}$	$\boxed{2}$ 1	$\boxed{9}$	$\boxed{7}$	1
$\boxed{7}$ 1	$\boxed{8}$	$\boxed{5}$	$\boxed{6}$	1
$\boxed{3}$	$\boxed{3}$	$\boxed{4}$	$\boxed{1}$ 1	1
$\boxed{7}$	$\boxed{5}$	$\boxed{2}$ 1	$\boxed{6}$	1
1	1	1	1	

cost = \$12

I need 4+4−1 = 7 basic variables. I have 4 so far. I'll choose three more (they'll be degenerate, but so be it.)

0	1		
1		0	
		0	1
		1	

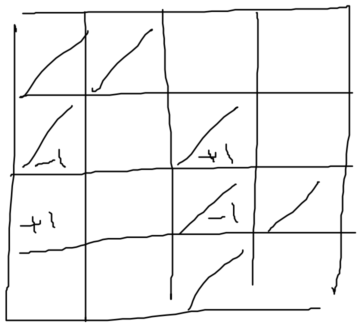
basic variables: $X_{11}, X_{12}, X_{21}, X_{23}, X_{33}, X_{34}, X_{43}$

I now consider the dual equations:

$$V_1 + W_1 = 4$$
$$V_1 + W_2 = 2$$
$$V_2 + W_1 = 7$$
$$V_2 + W_3 = 5 \quad \Rightarrow \quad V_1 = 0 \quad W_1 = 4$$
$$V_3 + W_3 = 4 \qquad V_2 = 3 \quad W_2 = 2$$
$$V_3 + W_4 = 1 \qquad V_3 = 2 \quad W_3 = 2$$
$$V_4 + W_3 = 2 \qquad V_4 = 0 \quad W_4 = -1$$

4 6	2 1	9 -7	7 -8		0
7 1	8 -3	5 0	6 -4		3
3 3	3 -1	4 6	1 1		2
7 -3	5 -3	2 1	6 -7		0
4	2	2	-1		

X_{31} is an entering variable that will yield a decrease in cost.



we see that x_{31} can't actually increase. If x_{31} increases, x_{33} has to decrease $\Rightarrow$ x_{33} would go negative since $x_{33} = 0$ right now.

$\Rightarrow$ we are at an optimal solution! Its cost is \$12.

<u>#12</u> supply $= 75 + 50 + 60 = 185$

demand $= 45 + 50 + 25 + 50 = 170$

demand $<$ supply, so we create a dummy warehouse as in #8.

$\boxed{5}$	$\boxed{6}$	$\boxed{1}$	$\boxed{4}$	$\boxed{0}$	75
$\boxed{2}$	$\boxed{9}$	$\boxed{7}$	$\boxed{5}$	$\boxed{0}$	50
$\boxed{8}$	$\boxed{5}$	$\boxed{8}$	$\boxed{7}$	$\boxed{0}$	60
45	50	25	30	15	

Apply Vogel's method, I find an initial basic feasible solution of

[5]	[6] 5	[7] 20	[4] 50	[0]	75
[2] 45	[9]	[7] 5	[5]	[0]	50
[8]	[5] 45	[8]	[7]	[0] 15	60
45	50	25	50	15	

Cost = \$720

$V_1 + W_2 = 6$

$V_1 + W_3 = 7$

$V_1 + W_4 = 4$

$V_2 + W_1 = 2$

$V_2 + W_3 = 7$

$V_3 + W_2 = 5$

$V_3 + W_5 = 0$

$\rightarrow$

$V_1 = 0$ $\quad W_1 = 2$

$V_2 = 0$ $\quad W_2 = 6$

$V_3 = -1$ $\quad W_3 = 7$

$W_4 = 4$

$W_5 = 1$

$\boxed{5}$ -3	$\boxed{6}$ 5	$\boxed{7}$ 20	$\boxed{4}$ 50	$\boxed{0}$ 1	75	0
$\boxed{2}$ 45	$\boxed{9}$ -3	$\boxed{7}$ 5	$\boxed{5}$ -1	$\boxed{0}$ 1	50	0
$\boxed{8}$ -7	$\boxed{5}$ 45	$\boxed{8}$ -2	$\boxed{7}$ -4	$\boxed{0}$ 15	60	-1
45	50	25	50	15		
2	6	7	4	1		

If X_{15} enters:

X_{12} departs,

save \$5

	-1			+1
	+1			-1

	-1	+1		
		-1		+1
	+1			-1

X_{25} enters

$\Rightarrow X_{12}$ departs save \$5.

Either way I save the same amount. I'll take X_{15} entering

		20	50	5
45		5		
	50			10

$\boxed{\text{Cost} = \$715}$

$V_1 + W_3 = 7$

$V_1 + W_4 = 4$

$V_1 + W_5 = 0$

$V_2 + W_1 = 2$

$V_2 + W_3 = 7$

$V_3 + W_2 = 5$

$V_3 + W_5 = 0$

$\Rightarrow$

$V_1 = 0$ $\quad W_1 = 2$

$V_2 = 0$ $\quad W_2 = 5$

$V_3 = 0$ $\quad W_3 = 7$

$W_4 = 4$

$W_5 = 0$

test for optimality:

$\boxed{5}$ -3	$\boxed{6}$ -1	$\boxed{7}$ 20	$\boxed{4}$ 50	$\boxed{0}$ 5	75	0
$\boxed{2}$ 45	$\boxed{9}$ -4	$\boxed{7}$ 5	$\boxed{5}$ -1	$\boxed{0}$ 0	50	0
$\boxed{8}$ -6	$\boxed{5}$ 50	$\boxed{8}$ -1	$\boxed{7}$ -3	$\boxed{0}$ 10	60	0
45	50	25	50	15		
2	5	7	4	0		

optimal!