

1 What is a *set*?

A *set* is a well-defined collection of things. “Well-defined” means that it must be possible to determine with certainty whether a given thing is in the set or not. The “things” in a given set may be objects of any kind, including symbols or even other sets, and are usually called *elements* of the set.

A set could be thought of as a container used to store certain objects. To decide whether an object is in the set or not, you would open the container and take a look.

Example. The following are examples of sets:

- The letters p, q, and r.
- All undergraduate students at University of Toronto.
- The students who are taking this class and who are named “John”.
- The set consisting of the even numbers and the number 17.
- The set of people who are related to you either by blood or marriage.

1.1 How to define a set?

If we are to work with sets, we need ways of defining and naming certain specific examples of sets. Here are some common ways of doing so.

1.1.1 (List the elements). One way to define a set is to simply list the elements. For example, the set of letters from a to f inclusive could be written using braces like so:

$$\{a, b, c, d, e, f\}.$$

When we define a set like this, we often want to give it a name, i.e. a symbol which refers to it. To give the set above the name “*S*”, we would write

$$S = \{a, b, c, d, e, f\}.$$

When naming sets, it is common to use capital letters; lower case is usually used for elements of sets.

Another useful symbol is “ \in ”, which means “is an element of”. If x is an element of the set X , then we write

$$x \in X.$$

1.1.2 (Give membership rules). Another way to define a set is to give a description of the set which determines whether an object is in the set or not.

For example, the description “the set of numbers from one to six inclusive” explains that in order to be in the set, one must first be a number, and also one must be between one and six. More compactly, we would write

$$X = \{x : x \text{ is an integer and } 1 \leq x \leq 6\}. \quad (1)$$

Note that we snuck in the new symbol “:”, which in this context means “such that”. Line (1) would be read aloud in the following way:

“ X is the set of elements x , such that x is an integer and is between 1 and 6, inclusive.”

1.1.3 (Grow the set organically). Yet another way to define a set is to start with certain elements (like seeds), and then explain a method for generating new elements from known elements. This produces a set consisting of elements which can be obtained from the seeds by applying the method a number of times.

For example, we could define a set by starting with the people in the class (the seeds) and then grow the set by the following method: if x is in the set, then any person who has ridden a bus with x is also in the set.