

# MAT 332HF Assignment 1

Due in .pdf, .jpg or .jpeg format on Quercus by 3 pm Thurs 20 Sept 2018

Here and hereafter, ‘West’ refers to problems from the required textbook *Introduction to Graph Theory. 2nd ed* by Douglas B West.

READ Chapter 1.1-1.2 by Thurs Sept 13 and Chapter 1.3-1.4 by Thurs Sept 20. We plan to cover Chapters 1-7 of the book in order, skipping all subsections marked optional, at a pace of slightly faster than two lectures per section.

Also, make sure you are familiar with the expected Mathematical Background from Appendix A.

WRITE UP solutions to the following problems, as succinctly and neatly as you can, and turn in on Quercus by 3 pm Thurs 20 Sept 2018 in either .pdf, .jpg or .jpeg format.

West §1.1 # 14, 34, 38; §1.2 # 20, 25, 38 §1.3 # 20, 25 (see below) plus

A. What is the smallest number of people needed at a party to ensure that either (a) three of the people are mutual acquaintances or (b) three of the people are mutual strangers? Prove it.

1.1.14 Prove that removing opposite corner squares from an  $8 \times 8$  checkerboard leaves a subboard that cannot be covered with domino tiles (i.e. 1 by 2 and 2 by 1 rectangles). Using the same argument, make a general statement about all bipartite graphs.

1.1.34 Decompose the Peterson graph into three connected subgraphs that are pairwise isomorphic. Also decompose it into copies of  $P_4$ .

1.1.38 Let  $G$  be a simple graph in which every vertex has degree 3. Prove that  $G$  decomposes into claws if and only if  $G$  is bipartite.

1.2.20 Let  $v$  be a cut-vertex of a simple graph  $G$ . Prove that  $\bar{G} \setminus v$  is connected.

1.2.25 Use ordinary induction on the number of edges to prove that absence of odd cycles is a sufficient condition for a graph to be bipartite.

1.2.38 Prove that every  $n$  vertex graph with at least  $n$  edges contains a cycle.

1.3.20 Count the cycles of length  $n$  in  $K_n$  and the cycles of length  $2n$  in  $K_{n,n}$ .

1.3.25 Prove that every cycle of length  $2r$  in a hypercube is contained in a subcube of dimension at most  $r$ . Can a cycle of length  $2r$  be contained in a subcube of dimension less than  $r$ ?