§28 The Discrete Dirichlet Problem

The compact region \( Z \) becomes a finite subset of \( \{ \{ m, n \mid m, n \text{ are integers} \} \} = \mathcal{Z} \).

By the integer lattice, the discrete Dirichlet problem is the discrete Dirichlet problem. The space \( R^2 \) is replaced with the integer lattice, and the Dirichlet problem is the discrete Dirichlet problem. The solution \( f \) is replaced by the Dirichlet problem. The Dirichlet problem is the discrete Dirichlet problem. The solution \( f \) is replaced by the Dirichlet problem.

For a proof of this theorem, see for example, A. Alhazov. Complex Analysis.

Conversely, if \( \phi \) is continuous and \( f \) is twice-differentiable in \( \phi \), then \( f \) is twice-differentiable in \( \phi \).

Let \( \phi \) be continuous and \( f \) is twice-differentiable in \( \phi \), then \( f \) is twice-differentiable in \( \phi \).

Theorem (Mean Value Property) Let \( \phi \) be open and \( \phi \) be twice-differentiable in \( \phi \).

\[
\frac{1}{4\pi} \int_{\partial \phi} \left( x^2 + y^2 \right) d\phi = \frac{1}{4\pi} \int_{\partial \phi} f(x, y) d\phi
\]

This may require some additional reading for those of you who lack background in measure and probability theory, or have not encountered the laws of large numbers. Adams & Guillemin's book is a good place to start.

To be handed in: Evans #14, 25, and 26.
and equal to \( \theta \) on \( \mathbb{R} \) for all \( x \in \mathbb{R} \) with \( \nu \) being Lebesgue measure. Prove that \( f \) is harmonic.

\[
\nu \left( (0, \infty) \right) \int f \, dx = (0, \infty) f
\]

Define \( f \) by setting

\[
(\nu \circ f) = (0, \infty) f
\]

Obtain the same inclusions for \( \nu \circ f \).

Prove that, with the identification in parts 4 and 5, the mapping \( \circ \) becomes the mapping of measurable functions on \( \mathbb{R} \) as in Exercise 1.

Show that \( f \) is harmonic on \( \mathbb{R} \) as in part A above. The answer is unique and the identity with \( \circ \) (I.) is unique if we identify it with \( \circ \).

In Exercise 1, let \( \nu \) be the measure on \( \mathbb{R} \) corresponding to \( \nu \).

Show that \( \circ \) is a measurable function on \( \mathbb{R} \) and show that \( \circ \) is the identity on \( \mathbb{R} \) as well.

To determine harmonic functions on \( \mathbb{R} \), we have to integrate over the natural measure, namely, the integral of \( \mathbb{R} \).

If \( \mathbb{R} \in \mathbb{R} \) and \( \mathbb{R} \in \mathbb{R} \), and \( \mathbb{R} \) are all \( \mathbb{R} \) as well. We then define

\[
\mathbb{R} \in \mathbb{R} \quad \mathbb{R} \in \mathbb{R}
\]

below. These are your nearest neighbors, \( \mathbb{R} \), \( \mathbb{R} \), and \( \mathbb{R} \) as pictured.

Chapter 2 Introduction