

These homework problems are meant to expand your understanding of what goes on during class. Any you turn in will be graded and returned to you. Answers may or may not be posted on the web, depending on demand.

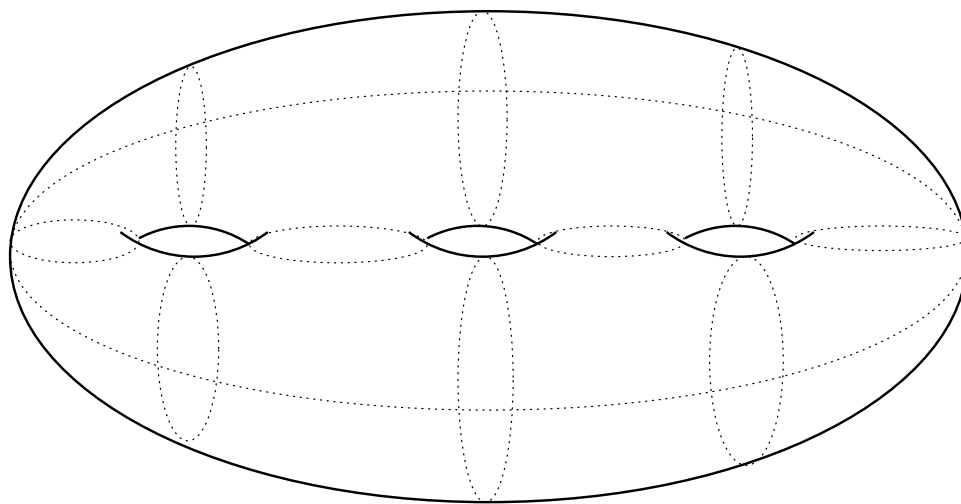
1. Finish the proof in class that there are only five Platonic solids (with Euler number  $\chi = v - e + f = 2$ ). That is, given that

$$4r = v(2d + 2r - dr)$$

where  $d$  is the common degree of all the vertices, and  $r$  is the common number of edges per face, prove that  $(d - 2)(r - 2) < 4$ .

**Hint:** All we really need from the equation above is that  $2d + 2r - dr > 0$ .

2. (a) Calculate  $\chi(P^2 \# P^2 \# P^2 \# P^2)$  by drawing the same sort of picture we had in class.  
 (b) Describe a cell division of  $P^2 \# \cdots \# P^2$  ( $n$  copies) in terms of  $v$  (the number of vertices),  $e$  (the number of edges), and  $f$  (the number of faces).  
 (c) Use part (b) to calculate  $\chi(P^2 \# \cdots \# P^2)$  ( $n$  copies).
3. For this problem, let  $(T^2)^n = T^2 \# \cdots \# T^2$  ( $n$  copies).  
 (a) Calculate  $\chi((T^2)^3)$  by drawing a cell division of  $(T^2)^3$  as shown as dotted lines in the crude diagram below:



(There should be eleven dotted ellipses indicating cuts.)

- (b) Calculate  $\chi((T^2)^4)$  by drawing a similar cell division of  $(T^2)^4$ .
- (c) How does adding an extra torus affect  $v$ ,  $e$ , and  $f$ ?
- (d) Use your answer to part (c) to write down  $v$ ,  $e$ , and  $f$  for a similar cell division for  $(T^2)^n$ .
- (e) Use the previous question to calculate  $\chi((T^2)^n)$ .