These solutions are written at the request of students. Please let me know if you don't understand these solutions; I'm happy to expand on them if necessary.

7. Show that if $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is an element of G, then

$$A^{-1} = \begin{bmatrix} \frac{d}{ad-bc} & -\frac{b}{ad-bc} \\ -\frac{c}{ad-bc} & \frac{a}{ad-bc} \end{bmatrix}$$

is its inverse. Again, you must show both that $A^{-1} * A = I$ and $A * A^{-1} = I$. Also, you should show that A^{-1} is an element of G.

Solution: First we show that the matrix A^{-1} defined above is an element of G. That is, we show that

$$\left(\frac{d}{ad-bc}\right)\left(\frac{a}{ad-bc}\right) - \left(-\frac{b}{ad-bc}\right)\left(-\frac{c}{ad-bc}\right) \neq 0.$$

This simplifies very quickly:

$$\left(\frac{d}{ad-bc}\right)\left(\frac{a}{ad-bc}\right) - \left(-\frac{b}{ad-bc}\right)\left(-\frac{c}{ad-bc}\right) = \frac{ad-bc}{(ad-bc)^2} = \frac{1}{ad-bc}.$$

Since $ad - bc \neq 0$ (since A is an element of G), this term is defined but non-zero.

Now we compute the product $A^{-1} * A$ and show that this is $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. The computation isn't pretty, but it follows from the definition of the product:

$$\begin{aligned} A^{-1} * A &= \begin{bmatrix} \frac{d}{ad-bc} & -\frac{b}{ad-bc} \\ -\frac{c}{ad-bc} & \frac{a}{ad-bc} \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \\ &= \begin{bmatrix} \frac{d}{ad-bc} \cdot a - \frac{b}{ad-bc} \cdot c & \frac{d}{ad-bc} \cdot b - \frac{b}{ad-bc} \cdot d \\ -\frac{c}{ad-bc} \cdot a + \frac{a}{ad-bc} \cdot c & -\frac{c}{ad-bc} \cdot b + \frac{a}{ad-bc} \cdot d \end{bmatrix} \\ &= \begin{bmatrix} \frac{ad-bc}{ad-bc} & \frac{bd-db}{ad-bc} \\ \frac{-ac+ac}{ad-bc} & \frac{-cb+ad}{ad-bc} \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I. \end{aligned}$$

The product $A * A^{-1}$ is similar.