SOAR Math Course **Homework Seven** Spring, 2003

These homework problems are meant to expand your understanding of what goes on during class. Any you turn in will be graded and returned to you. Answers may or may not be posted on the web, depending on demand.

1. Use Farey series to approximate $e \approx 2.718281828$. (Rather, use Farey series to approximate $e - 2 \approx 0.718281828$.) What is the error in this approximation?

2. Test each of the following numbers for divisibilty by: 2^k, 3^k, 5^k, 7, 11, and 101:
   (a) $741,954,888$
   (b) $1,039,715,607$
   (c) $44,774,865,619$

3. The number $101,617$ is the product of two "large" primes. Use Fermat's method to compute these two primes. (Hint: $\sqrt{101,617} \approx 318.8$.)

4. (a) One of the conditions for divisibilty relies on the fact that
   $$10x + y \equiv 0 \bmod 7 \qquad \text{if and only if} \qquad x - 2y \equiv 0 \bmod 7$$
   Prove this fact.
   (b) In class I erroneously claimed that
   $$10x + y \equiv x - 2y \bmod 7$$
   for any x and y. Prove, by example, that this is not true.
   (c) For what values of a does
   $$10x + y \equiv a \bmod 7 \qquad \text{if and only if} \qquad x - 2y \equiv a \bmod 7$$
   hold for all x and y?

5. [Anton's Question] Recall that the condition for divisibility by 11 is that 11 divides the alternating sum of the digits. Anton asks: Is the alternating sum of the digits of 11^k always zero?