SOAR Math Course **Homework One** Spring, 2003

These homework problems are meant to expand your understanding of what goes on during class. Any you turn in will be graded and returned to you. Answers may or may not be posted on the web, depending on demand.

1. Find the greatest common divisor (gcd) of the following pairs of integers.
   (a) 63 and 91
   (b) 207 and 156
   (c) Make up your own pair of integers.

2. Solve the following Diophantine equations for integer values of x and y. (At least one of these is *not* solvable, so you should check that it will be.)
   (a) $63x + 91y = 42$
   (b) $63x + 91y = 40$
   (c) $207x + 155y = 10$
   (d) Use your integers from 1(c) to construct an equation that you can solve. Now solve the equation.

3. Let a and b be positive integers with $\gcd(a, b) = 1$. Suppose $ab = x^2$ for some positive integer x. Must a and b be perfect squares?

4. This exercise will show give a general construction of *all* primitive Pythagorean triples
   (a) Find values of r and s so that the triple $(2rs, r^2 - s^2, r^2 + s^2)$ reproduce the Pythagorean triples $(4, 3, 5)$, $(12, 5, 13)$, and $(24, 7, 25)$. (I've written them this way so that a is even.)
   (b) Can both r and s be even? Odd?
   (c) Suppose (a, b, c) is a primitive Pythagorean triple with a even. Show that $\gcd(b + c, c - b) = 2$.
   (d) Show that $(b + c)/2$ is a square, as is $(c - b)/2$. (Use the previous problem.)
   (e) Conclude that *any* primitive Pythagorean triple can be written as $(a, b, c) = (2rs, r^2 - s^2, r^2 + s^2)$ where $r^2 = (b + c)/2$ and $s^2 = (c - b)/2$.

5. This exercise will show that there are infinitely many primitive Pythagorean triples.
   (a) Let (a, b, c) be a primitive Pythagorean triple with a even. Show that $(2ab, b^2, c^2 + a^2)$ (or, written another more helpful way, $(2ab, c^2 - a^2, c^2 + a^2)$) is also a Pythagorean triple.
   (b) Show that the new triple constructed in part (a) is primitive.
   (c) Argue that, because $c^2 + a^2 > c$, this implies that there are infinitely many primitive Pythagorean triples.

6. Let (a, b, c) be a Pythagorean triple, so $a^2 + b^2 = c^2$. Can a and b both be perfect squares? That is, can $a = x^2$ and $b = y^2$, so $x^4 + y^4 = c^2$? (This may be too difficult for this point in the class, but the *question* fits in even if the solution doesn't.)