Week 2: May 19th - May 25th

Suggested Problems

Problems you may find instructive, or that I find interesting.

§2.1 #27, 30, 32, 40, 45, 46 & 47
(Beware that #49 is misprinted.)

§2.2 #35, 39, 41, 46, 51, 52, 54 & 61(B)

§2.3 #20, 25, 29, 30, 43, 44, 45, 51, 57, 60 & 61
(#56 is also fun, if you have time. It is a bit like Russell's Paradox.)

§2.5 #5, 7, 13, 17, 31, 33, 43, 45, & 50
(If you want more practice with $\epsilon-\delta$, it is possible to prove #50 without the Squeeze Theorem.)

Assigned Problems

Due **May 26th**, in lecture.

1. Prove that **2.2.6**(ii) implies **2.2.6**(i); that is,

$$\text{“}\lim_{h\to 0} f(c+h) = L\text{”} \Longrightarrow \text{“}\lim_{x\to c} f(x) = L\text{”}.$$

   For clarity, use the notation ϵ, $\delta_{(ii)}$ and $\delta_{(i)}$.

2. Give an $\epsilon - \delta$ proof that,

$$\lim_{x\to 1^+} \frac{1}{x^2+2} = \frac{1}{3}.$$

   Hint: to find $x-1$, first write $\left|\frac{1}{x^2+2} - \frac{1}{3}\right|$ as a single fraction and factor the numerator.

3. Function $f(x)$ with domain $(-\infty, \infty)$ is *Lipschitz* if

$$|f(a) - f(b)| \leq |a-b| \quad \text{for any two values } a \text{ and } b.$$

   ~~Use the definition of continuity to prove that $f(x)$ is continuous at every point c.~~
   Prove that $\lim_{x\to c} f(x) = f(c)$ for every number c.