UNIVERSITY OF TORONTO

MAT 1300Y **Problem Set VI**

Due: Monday, April 9, 2007

1. Let $T = S^1 \times S^1$ denote the torus. Show that any map from S^2 to T is zero on reduced homology.

2. a) Show that T can be covered by 3 contractible open sets.
   b) Show that T cannot be covered by 2 contractible open sets.

3. Show that every self-map of $\mathbb{R}P^{2n}$ has a fixed point.

4. Show that $\mathbb{R}P^k$ is not a retract of $\mathbb{R}P^n$ for $n > k$.

5. Hatcher (page 260; #26)

6. a) Calculate $H^*(S^2 \times S^2)$ including the ring structure.
   b) Let $J_2(S^2)$ be the quotient space $J_2(S^2) = (S^2 \times S^2)/\sim$ where $(x, *) \sim (*, x)$. (This is called the James construction.) Calculate $H^*\big(J_2(S^2)\big)$ including the ring structure.
   c) Use the results of part (b) to show that $J_2(S^2)$ cannot be a manifold.

7. Show that there is no orientation-reversing self-homotopy-equivalence of $\mathbb{C}P^{2n}$.

8. (Qualifying Exam; Sept. 2003, # 6) Let $f : M \to N$ be a continuous map between compact oriented manifolds of dimension n, and assume that the induced map $f^* : H^n(N) \to H^n(M)$ is an ismorphism. Show that $f^* : H^p(N) \to H^p(M)$ is injective for all p.