UNIVERSITY OF TORONTO

MAT 1300Y **Problem Set V**

Due: March 9, 2007

1. (page 156; #9b)
2. (page 156; #10)
3. (page 156; #11)
4. (page 156; #12)
5. (page 53; #9)
6. (a) (page 157; #21)

   (b) (page 157; #20)
7. (page 158; #29)
8. Let $\phi : C \to D$ be a chain map. If ϕ is an injective map then we can form a quotient complex which fits into a long exact sequence with $H_*(C) \xrightarrow{\phi} H_*(D)$. It is sometimes convenient to have this in the case when ϕ is not an injection. To accomplish this, we construct a chain complex $\tilde{D}$ and a chain homotopy equivalence $j : D \to \tilde{D}$ together with an injection $i : C \to \tilde{D}$ such that $i \simeq j \circ \phi$ and $\phi = k \circ i$, where k is a chain homotopy inverse to j.
   a) Define $\tilde{D}$ by $\tilde{D}_p = C_p \oplus D_p \oplus C_{p-1}$. Define $\tilde{\partial} : \tilde{D}_p \to \tilde{D}_{p-1}$ by $\tilde{\partial}(c_p, d_p, c_{p-1}) = \big(\partial c_p - c_{p-1}, \partial d_p + \phi(c_{p-1}), -\partial c_{p-1}\big)$ for $c_p \in C_p$, $d_p \in D_p$, and $c_{p-1} \in C_{p-1}$. Verify that $(\tilde{D}, \tilde{\partial})$ forms a chain complex.
   b) Define $i : C \to \tilde{D}$ and $j : D \to \tilde{D}$ by $i(c) = (c, 0, 0)$ and $j(d) = (0, d, 0)$. Verify that i and j are chain maps.
   c) Show that $i \simeq j \circ \phi$.
   d) Find a chain map $k : \tilde{D} \to D$ having the properties that k is a chain homotopy inverse to j and that $\phi = k \circ i$.

   Remark: $\tilde{D}$ is called the *Algebraic Mapping Cylinder* of ϕ.