UNIVERSITY OF TORONTO

MAT 1300Y **Problem Set IV**

Due: Friday, February 9, 2007

1. A chain complex C is called indecomposable if there do not exist nonzero chain complexes C', C'' such that $C \cong C' \oplus C''$. Find the indecomposable positively graded free chain complexes of finite type. (That is, C_n is a finitely generated free abelian group for all n with $C_n = 0$ for $n < 0$.) Show that if C is a positively graded free chain complex of finite type then C is a direct sum of indecomposable chain complexes.

2. Give an example of two spaces which have the same homology groups but are not homotopy equivalent.

3. Compute the map induced on homology by the following maps:
   a) The inclusion map $S^1 \vee S^1 \to S^1 \times S^1$.
   b) $i_1 : S^1 \to S^1 \times S^1$, $i_1(x) = (x, *)$.
   c) $\pi_1 : S^1 \times S^1 \to S^1$, $\pi_1(x, y) = x$.
   d) $\Delta : S^1 \to S^1 \times S^1$, $i_1(x) = (x, x)$.
   e) $m : S^1 \times S^1 \to S^1$, $m(x, y) = xy$.
   f) $\mathrm{sq} : S^1 \to S^1$, $\mathrm{sq}(x) = x^2$.

   Note: While the answers can be expressed as certain maps between abstract groups (e.g. multiplication by five: $\mathbb{Z} \to \mathbb{Z}$), a preliminary step is to choose an identification of the relevant homology groups with the appropriate abstract group. Thus depending on your idenfications, some of the above will become definitions rather than computations.

4. Recall that a Möbius band is the locus of a stick whose centre is attached to a circle and then gradually rotated as it is moved around the circle until after one revolution it is upside down from its original position. Alternatively, it can be described as the space formed from a rectangle by gluing one pair of opposite sides together after inverting one of the sides. Note that the boundary of a Möbius band is homeomorphic to S^1. Let X be the space obtained by cutting a disk out of a torus and then gluing on a Möbius band M by "attaching" its boundary to the boundary of the hole created. Calculate the homology groups of X.

5. Find the homology of the space formed by taking two Möbius bands and "gluing them together" by identifying their boundaries.

6. (page 131;#17b)

7. (page 155; #2)

8. (page 155; #4)